Investigating Clustered States in the $^9$Be Nucleus through Inelastic Alpha Particle Scattering

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Abstract

An experiment was performed on the Birmingham MC40 cyclotron to study the $^9$Be$(\alpha, \alpha)^9$Be$^*$ → $3\alpha + n$ breakup reaction, in order to investigate the structure of the $^9$Be nucleus. The analysis has provided very strong evidence for new, previously unreported states in $^9$Be with energies in the region of 9-13 MeV. Nuclear excitation was achieved through fixed target inelastic $\alpha$ particle scattering. The breakup yields for each of the $^9$Be decay channels ($^8$Be$_{\text{(ground state)}}$ + $n$ and $^8$Be$_{\text{(2+ state)}}$ + $n$) were subsequently measured over an energy range between 0 and 25 MeV. Dalitz plot analyses allowed two $^9$Be excitation spectra to be calculated, specific to each breakup channel. A fit of all known energy states to these spectra provides a poor fit for excitation energies around 9-13 MeV. Introducing two new states to this region provided an excellent fit to the data, though gave suspiciously low $\chi^2$ values. The parameters defining the two new states were consistent across both fits. State 1 was found to have an energy of 9.74 / 9.96 MeV and a width of 2.64 / 3.44 MeV. State 2 was found to have an energy of 12.75 / 12.79 MeV and a width of 1.25 / 1.63 MeV. The spins and parities of the new states were determined through the comparison of their breakup yields with those predicted by centrifugal barrier penetrability calculations. The new states were found to be either (3/2$^+$ or 5/2$^+$) and (7/2$^+$ or 9/2$^+$) respectively. However, it is clear that a more rigorous assessment of the errors is required to confirm these assignments. This analysis method was successfully tested on known states at 11.2 and 11.8 MeV. The findings agree with previous assignments of the spins and parities as 7/2$^+$ and 5/2$^+$ respectively.

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1 Introduction

Cluster models predict that, instead of being thought of as a homogeneous sphere of protons and neutrons, the nucleus is best described as a collection of clusters where a cluster is defined as a localised collection of nucleons. In some cases, theoretical models assuming this structure have very successfully reproduced the energy levels of certain nuclei [1]. However, since there is no single theory that describes the structure of the atomic nucleus, it is still the case that many nuclear properties can be only be accurately determined experimentally. Such an experimental investigation into the energy levels of \(^{9}\text{Be}\) is the focus of this project, in which the aim is to populate and measure excited states within \(^{9}\text{Be}\) through inelastic \(\alpha\) particle scattering \(^{9}\text{Be}(\alpha, \alpha)^{9}\text{Be}^*\) reactions performed on the Birmingham MC40 cyclotron.

This report shall firstly cover the basic theory of nuclear clustering along with the proposed formation of rotational bands within clustered nuclear structures. Next, a more in depth look at the theorised structure of \(^{9}\text{Be}\) is taken along with the motivation for studying this nucleus in particular. The reaction details are discussed along with a description of the experimental setup used to induce and measure these reactions. The primary data analysis stages of reconstructing raw excitation spectra for \(^{9}\text{Be}\) are covered in depth, along with the secondary data analysis stages which involve an approximate Breit-Wigner peak-fitting to extract information about the measured energy states.

2 Theory and Experimental Aims

2.1 Nuclear Clustering and Rotational Bands

An important concept within the theory of nuclear clustering is that of rotational bands. Particular cluster configurations within a nucleus correspond to a particular moment of inertia of the system, \(I\). Certain excited states can be populated by providing the system with higher values of angular momentum. Classically, the energy of a rotating body of moment of inertia, \(I\), angular frequency, \(\omega\), and angular momentum, \(L\), is given by the following formulae:

\[
E_{\text{Rot}} = \frac{1}{2} I \omega^2 \quad \text{(1)}
\]

\[
= \frac{L^2}{2I} \quad \text{(2)}
\]

Translating this equation into the quantum mechanical domain involves the introduction of the total angular
momentum operator $\hat{J}^2$. Assuming that the excitation energy of a nucleus is purely rotational, the eigenvalues for the energies of the states are then given by the simple formula [2]:

$$ E_x = \frac{\hbar^2}{2I} j(j + 1) $$

(3)

The assumed fixed moment of inertia, $I$, then ensures that energy levels corresponding to a particular cluster configuration are proportional to $j(j + 1)$, and so form a 'rotational band.'

Figure 1: a) Ikeda diagram illustrating that particular cluster degrees of freedom are unlocked as the nuclear excitation energy approaches the cluster decay threshold b) The proposed rotational band structure of $^{20}\text{Ne}$ [4]

As was first proposed by Ikeda [3], the total excitation energy of a clustered nucleus is, in fact, expected to be given by the sum of the rotational energy of the nucleus and an extra energy that is required to initially form the clustered structure. Ikeda approximated that particular cluster configurations form when the excitation energy of the nucleus approaches the cluster decay threshold $E_0$ as illustrated in the Ikeda diagram shown in figure 1 a). $E_0$ is defined as the mass difference between the ground state nucleus and the sums of the masses of each cluster component. Therefore the full form for the excitation of a clustered nucleus is given as:

$$ E_x = \frac{\hbar^2}{2I} j(j + 1) + E_0 $$

(4)

As an example, the appearance of rotational bands in the known energy levels of $^{20}\text{Ne}$ are shown in figure 1 b).
2.2 Structure of $^9$Be and Project Motivation

Possibly the simplest nuclear cluster structure is that of $^8$Be, which is believed to consist of two $\alpha$ particles in a dumbbell-type structure. On the simplest theoretical level, this structure emerges from simple deformed harmonic oscillator mean field calculations (Appendix A) at a 2:1 deformation. Furthermore, more complex Greens function Monte Carlo (GFMC) calculations which model simplified 2-body and 3-body nucleon-nucleon interactions without the need for a mean field also agree with a highly clustered ground state. The nuclear density projections predicted by these two approaches are shown in figure 2.

![Figure 2: $^8$Be nuclear density projections derived from a) deformed harmonic oscillator calculations [5] and b) more thorough GFMC calculations [6]](image)

Experimental measurements of the energy levels in $^8$Be are also in good agreement with a highly clustered structure. Figure 4 a) shows the energies of the first three states in $^8$Be plotted against their total angular momenta, forming the ground state rotational band. The linear relationship agrees with that predicted by equation 4 and the moment of inertia calculated from the gradient of the best fit is in good agreement with the classical analogue of a 2$\alpha$ structure.

Given that $^8$Be has a lifetime of approximately $10^{-16}$ seconds and that $^9$Be is stable, one way to picture the $^9$Be nucleus is as a "nuclear molecule." This model predicts that the extra valence neutron introduced into the $^8$Be system is exchanged between two $\alpha$ particle cores to increase the overall binding energy of the nucleus.
The experimentally measured energy levels and assigned rotational bands of $^9$Be are shown in figure 4 b). This band structure is in excellent agreement with the predictions of several different theoretical approaches. The band heads are successfully predicted by the Nilsson model of the deformed shell model at a $2:1$ deformation (figure 4 a) and the band structure is reproduced through more complicated GMC calculations (figure 4 b). Furthermore, calculations performed within the framework of the AMD produce projections of the nuclear density consistent with the idea of a neutron being shared between two $\alpha$ particle cores, as shown in figure 5.
The nucleon density distributions of the $^9$Be ground state calculated within the framework of the AMD [9].

Theory predicts that a fourth state belonging to the $3/2^-$ rotational band should lie at an energy of approximately $11.12 \pm 0.34$ MeV with a spin and parity of $9/2^-$. This state is circled in green in figure 4 b). The aim of this project was to measure this state directly. One explanation for this missing state could be a possible mislabelling of the spin of a known state at $11.283 \pm 0.024$ MeV. There has been some dispute in the past regarding the measured spin and parity of this state [10] [11] [12]. Therefore, one aim of the project was to measure the spin and parity of this state to determine if this should be assigned to the $3/2^-$ band. Another explanation could be that the missing state has not before been measured due to it’s predicted broad width meaning that it is easily concealed by a background signal.

2.3 Reaction Details and Kinematics Calculations

The type of reaction examined during the experiment was that of inelastic $\alpha$ particle scattering, whereby a beam of $\alpha$ particles from the Birmingham MC40 cyclotron strike a solid $^9$Be foil target. A fraction of the centre of mass energy of the system is imparted to the internal excitation of the recoiling $^9$Be which then goes on to decay via one of the breakup modes ($^8$Be + $n$) or ($^5$He + $\alpha$) → $\alpha + \alpha + n$. This process is shown schematically in figure 6.
Detection of the scattered \( \alpha \) particle allows a reconstruction of the excitation energy of the \(^9\text{Be}\). Before the initial collision, the energy in the system is just that of the incident beam particle:

\[
E_{\text{before}} = E_{\text{beam}} + Q_{gs} \quad (0 \text{ MeV}) \tag{5}
\]

After the collision the total energy is the kinetic of the scattered beam and recoiling \(^9\text{Be}^*\), in addition to the internal excitation energy of the recoiling \(^9\text{Be}^*\):

\[
E_{\text{after}} = E_k(\alpha_{\text{scatt}}) + E_k(\text{Be}^*) + E_x \tag{6}
\]

Energy conservation between the initial and final states gives:

\[
E_x = E_k(\alpha_{\text{scatt}}) + E_k(\text{Be}^*) - E_{\text{beam}} - Q_{gs} \tag{7}
\]

As shown in Appendix C, knowing the energy of the scattered \( \alpha \) particle and its point of detection allows a calculation of its three cartesian momentum components. Therefore, by knowing the beam energy along with the energy of the scattered \( \alpha \) particle, momentum conservation allows the kinetic energy of the recoiling \(^9\text{Be}\) to be determined. Equation 7 can therefore be evaluated through only the measurement of the scattered \( \alpha \) particle. This fact will be key to the primary analysis described in section 5.
3 Experimental Apparatus

3.1 Beamline Overview

The general experimental setup available on the cyclotron beamline is shown in figure 7. A beam of $\alpha$ particles exit the cyclotron by magnetic deflection and enter the beamline. Here they travel down an evacuated beam pipe at a pressure of $10^{-4}$ torr to the cylindrical experimental chamber.

![Figure 7: General overview of the experimental setup on the beamline](image)

Once inside the chamber, the beam undergoes two stages of collimation. The first stage forces the beam to pass through a 2mm diameter circular collimator, closely followed by a 7mm diameter "anti-scatter" collimator. The whole of the collimator setup is shielded by lead bricks and L-shaped steel blocks to further prevent any scattered particles from reaching the detectors. The two collimator apertures are held on an adjustable stand which was aligned with the use of a laser beam, which simulates the direction that the cyclotron beam travels during experiments. Once collimated, the beam can collide with the target which, if solid, is fixed to a remotely controlled holder attached to the chamber lid. This control allows multiple targets to be used in a single experiment, without the need to repressurise the reaction chamber. Upon striking the target the incident particles cause nuclear reactions, the products of which are detected by an array of four charged particle semiconductor strip detectors as described in section 3.2.

3.2 Detector Setup

3.2.1 Silicon Charged Particle Detectors: Theory and Specification

The charged reaction products are detected by an array of four "Micron" position sensitive silicon strip detectors (PSSSD) [13] of thicknesses 0.5 mm and 1 mm. The active area of each detector is 5cm x 5cm and is separated
3.2 Detector Setup

into a front and back face. These are respectively split into 16 vertical and horizontal strips, where each
strip behaves as an individual detector in its own right. When a charged particle traverses the detector, the
crossing point of the vertical front and horizontal back strips that the particle is detected by provides the spatial
resolution. A schematic diagram and photograph of a single detector are shown in figure 8.

The interface between the front and back faces forms a depletion region, in which the holes from the rear
P-type strips and electrons from the front N-type strips recombine. A 100V and 120V reverse bias is applied
across the 0.5mm and 1mm detectors respectively, meaning that the depletion region grows as shown in figure
9, creating a large charge-detection medium. The rear detector face is subject to a positive 100 or 120V DC
supply, whereas the front face is kept at 0V. When a charged reaction product passes through this region, it
excites a certain number of electron-hole pairs in the silicon.
In order to completely stop a several MeV particle, several million electron-hole pairs are produced. The large number of particles leads to excellent counting statistics and suggests a theoretically good energy resolution. The number of pairs produced is proportional to the energy of the incident quantum. These drift, under the influence of the reverse bias, to opposite sides of the detector to be collected by the electrodes. This induces a current pulse which exits each individual detector strip via a wire bond to a small circuit board. The integral of the current pulse provides the total charge collected and is therefore proportional to the incident particle energy.

### 3.2.2 Spatial Arrangement

The spatial arrangement of the detectors inside the chamber is shown in figure 10 with the position measurements detailed in table 1. The detectors were brought as radially close to the target as the setup would allow, with the purpose of increasing the overall efficiency of the setup. However, this has the undesired effect of worsening the position resolution of the detection, due to the fact that each detector strip subtends a larger solid angle with respect to the target.

The minimum angle covered with respect to the beam direction is approximately $\pm 10^\circ$ to ensure that no direct beam strikes the detectors. The outer detectors extend to a maximum angle of approximately $\pm 80^\circ$. The outer two detectors lie radially further inwards towards the target than the inner two. This permits a small
3.3 Processing Electronics

The overall aim of the processing electronic setup was to convert the raw analogue current pulse that exits the detector in the event of a charged particle interaction into a digital binary signal that can be inputted into the computer for analysis. The setup employed for this experiment also searches across all detector channels for coincident signals within a 500ns window, allowing detections to be grouped together into events, corresponding to a single nuclear reaction. This allows a selection of the type of reaction being recorded because as was mentioned in section 2.3, a three particle final state corresponds to a reaction involving $^9$Be.

A schematic diagram that shows the ordering of the electronic equipment is shown in figure 11. A 100/120V
3.3 Processing Electronics

power supply applies a reverse bias to each detector, reaching these via Amseytec MPR32 preamplifiers. The current pulses generated within the detectors due to charged particle interactions exit the detectors and are integrated by the preamplifiers into voltage pulses, which have heights proportional to the energy of the detected quanta. The strips on each detector behave as individual detectors and, therefore, the processing electronics must accommodate a total of 128 channels of data (32 strips x 4 detectors). This being the case, a total of 4 preamplifiers were used, each with two 16-channel input/output ports, corresponding to one for each face of each detector.

![Schematic diagram showing the ordering of the pulse processing electronics](image)

The outputs from the preamplifiers are carried to eight 16-channel CAEN N568 LC amplifiers via ribbon cables. Each amplifier corresponds to the pulses from either the front or back face of a detector. These are contained within the Wiener UEN05 crate which allows the amplification settings to be adjusted on the computer using the MIDAS software, further explained in section 3.4. To achieve full communication, the eight amplifiers were attached together in a daisy-chain fashion.

The overall setup is subject to a small amount of electronic noise, which if not accounted for, will result in false, low energy signals being measured. This is overcome through pulse height discrimination whereby only amplifier outputs corresponding to a "significant" pulse height are deemed suitable for further analysis. As explained in section 3.2, the passage of a charged particle through the detector should result in approximately the same amount of energy being detected on the front and back face of the detector. Therefore, pulse height discrimination only needs to be applied to the front face detector channels. Four discriminators were used to determine whether a significant signal had been measured, each corresponding to a single detector. These
discriminators are housed in a Wiener crate and can again be controlled from the computer using the MIDAS software. If a pulse entering the discriminator exceeds the preset threshold level, a logic pulse of approximate height 50 mV is outputted from the discriminator.

All discriminators are connected in a daisy-chain ensuring that the logic pulses from each discriminator are summed before entering the final discriminator module where multiplicity conditions are applied. A multiplicity condition was set in MIDAS for the detection of three coincident pulses within a 500ns time window. If the multiplicity conditions are satisfied, a trigger is sent from the final discriminator module to the ADC control unit, to signify that the current data from the amplifiers can be recorded onto the computer via the ADCs. The ADCs used are the Silena S9418 model, which use the Successive Approximations method to convert the raw analogue pulses from the amplifiers into binary signals. The ADC control unit is connected to the ADCs which are each connected together to allow control from the computer.

3.4 Data Acquisition and Analysis Software

3.4.1 MIDAS

Communication between the computer and electronic equipment is achieved via the MIDAS software. Here, certain settings of the amplifiers and discriminators can be configured along with where and when the data is written to disk.

3.4.2 Sunsort

Sunsort is a data analysis package designed by the CHARISSA collaboration. The software provides the opportunity to analyse data on an event by event basis, either from disk or during data acquisition. The working of the Sunsort framework is described within the online documentation as follows [14]:

"At the heart of the Sunsort package is a library of C functions which provides the user with facilities to access data ... and to decode this data into a simple array of ADC values. A user provided FORTRAN or C subroutine is linked to this library to form the core sort process. During sorting, the user’s subroutine is called once per event and is passed the array containing the ADC values. The user can call upon various Sunsort library utility routines in order to, amongst other things, increment spectra, perform windowing operations and filter selected events back to a storage medium."

A basic Fortran sort code was written by Professor Martin Freer which imports the raw ADC values for
each detector channel, on an event by event basis. This was built on throughout the project to perform the required primary data analysis as described later in section 5. The final sort code is shown in Appendix F. Separate codes were written to perform other standalone tasks. These are referenced throughout this report and included in the appendices.

3.5 Energy Calibration and Resolution Measurements

3.5.1 Energy Calibration

An energy calibration was required for each of the 128 detector channels in order to convert the raw output from the amplifiers into a value for the energy of the detected particle. This was partially achieved by exposing the complete detector array to a mixed Americium, Curium and Plutonium source, which decays via the emission of alpha particles at three main energies: 5.14, 5.49 and 5.70 MeV. The calibration is achieved by comparing the response of each detector to the actual energy of the α particles and determining the linear relationship between these two quantities. By exposing the detectors to the source for a suitable length of time, a raw energy spectrum for the source could be plotted for each detector channel. An example spectrum is shown in figure 12 a).

Figure 12: a) Three peaks fitted to a raw spectrum, corresponding to a single detector channel. b) A plot of peak centroid vs alpha particle energy.

Correlating the centroid of each peak in the spectrum with the corresponding α particle energy allows plots similar to that shown in figure 12 b) to be formed. As was mentioned in section 3.2, the detector output is directly proportional to the particle energy. Hence, a linear least squares fit to this allows a channel (X) to energy (E) conversion to be produced, of the form:
3.6 Detector Resolution Measurement

\[ E = mX + c \]  
(8)

Such a calibration and least squares fitting has been automatically implemented into the Fortran sort code. To achieve this, the peaks in the calibration \( \alpha \) spectra were still required to be fitted manually for each of the 128 detector channels using buffit. Figure 13 shows the responses of all detector channels before and after a full energy calibration has been applied.

![Energy intensity plots for each detector channel](image)

Figure 13: Energy intensity plots for each detector channel when a) No calibration has been applied. b) After the full energy calibration has been applied.

As shall be seen in section 3.6, only using low energy alpha particles for calibration leads to problems when extrapolating the line of best fit to higher energies. A possibly more robust calibration method would be to consider the elastic scattering of the beam from the target during the experiment (as seen later in section 5.2). The scattering kinematic lines can be used as a reference for calibrating the system. Time did not permit the investigation of this possibility.

3.6 Detector Resolution Measurement

A knowledge of the detector resolution is important because of the effects that this has on the appearance of the eventual \(^9\)Be excitation spectra, that result from the analysis described in section 5. An ideal detector has infinite resolution, where the energy of the particle incident upon the detector is identically equal to the value that is measured. Realistically this cannot be achieved and all detectors are subject to an \textit{intrinsic} resolution.
3.6 Detector Resolution Measurement

that depends on the exact detection mechanism employed. The silicon detectors used in this work rely on the excitation and subsequent detection of a certain number of electron-hole pairs within the detection medium. The number of detected electron-hole pairs, $N$, is subject to standard counting statistics, meaning that the detected energy follows a Poisson distribution about a mean. Therefore, the fractional resolution, $R$, is expected to follow a $1/\sqrt{E}$ dependence.

Each calibration energy spectrum such as that shown in figure 12 a) can be used to determine the detector resolution of each detector strip. Each $\alpha$ particle energy appears as a gaussian distribution in the high-$N$ limit, with a centroid $\bar{E}$ and a width given by the Full-Width Half-Maximum (FWHM). The absolute detector resolution is equal to the FWHM and the fractional detector resolution is defined as $\text{FWHM}/\bar{E}$. Measuring the centroid and FWHM of each of the three peaks in the calibration spectra and converting into units of energy (equation 12) allows the resolution of each peak to be calculated. The absolute and fractional resolution as a function of energy for a single detector strip are shown in figure 14.

![Figure 14: Plots of absolute and fractional intrinsic detector resolution as a function of particle energy](image)

These plots show that the absolute resolution is approximately 60 KeV, and remains approximately constant throughout this energy range, within errors. A least squares fit was performed (shown in figure 15) which gave an eventual resolution energy dependence of:

$$R = (1.2431 \pm 0.4113)E^{(-0.9193 \pm 0.5604)}$$  \hspace{1cm} (9)
3.6 Detector Resolution Measurement

Within large errors, this does follow the $E^{-1/2}$ dependence predicted by theory. However, for a 5 MeV detection, approximately 5 million electron-hole pairs should be excited within the silicon. For a detector dominated by counting statistics, the fractional resolution can be calculated to be $1/\sqrt{N} \approx 5 \times 10^{-4}$, almost two orders of magnitude smaller than the measured values in this energy region. In fact, the resolution is dominated by the baseline electronic noise in the system, which is expected to remain constant regardless of energy. Both the energy calibration and resolution were calculated for the example detector strip using a Matlab code which is shown in Appendix B.1.

However, this analysis only covers low energies. To extend this to the higher energies detected on the beamline a second approximate method was employed. For two perfectly calibrated front and back strips, upon detection of a particle, the same energy should be registered, only differing by an amount $\Delta E = E_f - E_b$ due to the finite detector resolution. To measure this, the data collected during the beam run was sorted using the code in Appendix B.2. For each particle detection the measurements of the particle’s energy on the front and back face are identified. The difference is calculated and this is plotted against the mean energy detected between the front and back face on a 2D intensity plot. This is shown in figure 16.
3.6 Detector Resolution Measurement

Figure 16 demonstrates that $\Delta E$ follows a spread of energies centred on zero. The width of the spread was calculated for a number of energies, by fitting a gaussian distribution to several vertical cuts of figure 16. This allows the following plots of absolute and fractional energy resolution to be calculated using the code shown in Appendix B.3.

Figure 17: a) Absolute resolution as a function of energy b) Fractional resolution as a function of energy

As higher energies are reached, the absolute detector resolution increases in a linear fashion. This can be explained by considering the imperfect energy calibration performed using the 3o source.
only calibrated at low energies, errors will be incurred when extrapolating the fit to higher energies, linearly amplifying any effects of the resolution as higher energies are reached. Therefore the calculated resolutions are not representative of the true resolution. The Matlab code fits a power law curve to the data and reveals the following relationship, which is fitted to the data in figure 18.

\[ R = (1.2431 \pm 0.4113)E^{(-0.7487 \pm 0.1264)} \]  

Figure 18: Least squares fits for the fractional resolution as a function of energy

What is being viewed here is an average resolution across all poorly calibrated detector strips. In order to perform a full analysis, a thorough calibration and separate resolution must be calculated for each strip individually.

4 Experimental Beam Runs

The data used during the final analysis was acquired during a single beam run with the beam time separated between reactions involving two targets. The first was $^9$Be which was required for the majority of the analysis and the second was $^{12}$C which was required for background suppression as is described in section 5.5.1. The beam run statistics are shown in table 2

<table>
<thead>
<tr>
<th>Target</th>
<th>Beam Exposure Time (Hours:Minutes:Seconds)</th>
<th>Amount of Data Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9$Be</td>
<td>1:41:55</td>
<td>11201</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>0:58:42</td>
<td>9699</td>
</tr>
</tbody>
</table>

Table 2: Details of the experimental beam run
5 Primary Data Analysis

The raw data recorded during the beam run was analysed using the Fortran sort code shown in Appendix F within the Sunsort framework. The overall aim of the primary data analysis is to convert the raw data recorded during the beam run into various excitation spectra for the target nuclei. This process can be separated into several event filtering and calculation steps that shall be described in more detail throughout the following sections.

5.1 Determination of the Energy and Momentum of the Final State Particles

The detector design as described in section 3.2 ensures that upon traversing the detector, the rear electrode of the detector detects the same amount of electrons as the front electrode detects holes. Therefore, it follows that upon striking a certain region of the detector, the energy detected on the front, vertical strip subtending this region of the detector should be equal to that detected in the corresponding horizontal rear strip, within the errors dictated by the detector resolution. This energy equality is the primary method used to match the front and back strips corresponding to an individual particle detection, therefore allowing the x-y position of the detection on each detector to be determined. The energy of the particle is arbitrarily taken as the energy detected by the front face. This energy matching process is shown in lines 4455 to 498 of the sort code in Appendix F.

Once the x-y position of the detection has been determined, the measurements shown in figure 10 can be used to calculate the horizontal and vertical angles of each detection with respect to the target and the beam direction as shown in figure 19. The full method of calculating these angles is explained fully in Appendix C, along with how they are used to calculate the three cartesian momentum components of a detected particle.

![Figure 19: Diagram showing the horizontal and vertical angles required to fully label the position of a particle detection within the detector array with respect to the initial beam direction](image-url)
5.2 Beam Energy Calculation

As described in section 2.3, an accurate knowledge of the beam energy is crucial for calculating the excitation energy of $^9$Be. To correctly identify this unknown parameter, a section of the sort code was written that plots the energy of each detected particle against its total angle from the beam, as shown in figure 20. This plot shows several strong kinematic lines from the detection of the scattered $\alpha$ particles. The parallel lines are due to elastic and inelastic scattering from states in $^9$Be and the uppermost two lines are due to elastic scattering from the contaminant $^{12}$C and $^{16}$O in the target.

To estimate the beam energy, theoretical kinematic lines were plotted on top of the data using the equations derived in Appendix D, for the ground state scattering reactions in $^9$Be, $^{12}$C and $^{16}$O. The beam energy used within the calculation was varied until the best fit to the data was found. This also provided an opportunity to slightly adjust the distances and angles of the detectors until the data fit to the kinematic lines, as these parameters were not originally measured to a high degree of accuracy. An optimal beam energy of 40.2 MeV was found.

![Figure 20: Fit of the theoretically derived kinematic lines to the data for a 40.2 MeV beam. Note that the measured values are lower than those predicted at large scattering angles, due to losses in the target.](image-url)
5.3 Sum Energy Spectra

The origin of the final state reaction products can be determined by constructing their sum energy spectrum. For each event that is read into the sort code, a condition is placed on the number of final state particles in an event. For example, when examining a reaction involving \(^9\)Be, three final state \(\alpha\) particles are expected through the overall breakup reaction:

\[
\alpha + ^9\text{Be} \rightarrow \alpha + \alpha + \alpha + n \quad Q = -1.574 \text{ MeV}
\]  

Using energy conservation between the initial and final state, the beam energy is given by the formula:

\[
E_{\text{beam}} = E_{\alpha_1} + E_{\alpha_2} + E_{\alpha_3} + E_n - Q
\]

The energy of the neutron reaction product due to a \(^9\)Be breakup is calculated using momentum conservation between the initial and final states. Taking the beam to be travelling only in the \(z\) direction, the neutron energy is determined as follows:

\[
\begin{align*}
    P_{x_n} &= -(P_{x_{\alpha_1}} + P_{x_{\alpha_2}} + P_{x_{\alpha_3}}) \\
    P_{y_n} &= -(P_{y_{\alpha_1}} + P_{y_{\alpha_2}} + P_{y_{\alpha_3}}) \\
    P_{z_n} &= P_{\text{beam}} - (P_{z_{\alpha_1}} + P_{z_{\alpha_2}} + P_{z_{\alpha_3}}) \\
    |P_n| &= \sqrt{P_{x_n}^2 + P_{y_n}^2 + P_{z_n}^2} \\
    E_n &= \frac{P_n^2}{2m_n}
\end{align*}
\]

Within the sort code, for each three-particle event, the energy of the beam is calculated using equation 12. Should the assumption that the three particles detected came from \(^9\)Be be correct then the calculated beam energy should be equal to the approximately 40.2 MeV beam energy determined in the previous section. Figure 21 shows the sum energy spectrum for 3-particle events.
5.4 Dalitz Plot Analysis

Figure 21: The sum energy spectrum for three particle events under the assumption that the reaction occurred with a \(^9\)Be nucleus in the target, with and without a fit.

The fit of a skewed gaussian with an exponential background to this spectrum in figure 21 gives that the spectrum peaks at \(403.79 \pm 0.29\) MeV, corresponding to a beam energy of 40.379 MeV. This is only slightly different to the 40.2 MeV beam energy that was approximated in section 5.2 and used in equation 17 to calculate the energy of the missing neutron. The FWHM of the peak was calculated as \(6.873 \pm 0.072\) MeV (\(\sigma = 2.924 \pm 0.031\) MeV). Most events within this peak correspond to a reaction involving \(^9\)Be. The peak is not sharp, however, and displays a long tail extending to high energies. This is due to the fact that the three final state particles measured may actually have been due to the beam interacting with a contaminant in the target such as \(^{12}\)C or \(^{16}\)O:

\[
\begin{align*}
\alpha + ^{12}\text{C} & \rightarrow \alpha + \alpha + \alpha + n & Q = -7.275 \text{ MeV} \\
\alpha + ^{16}\text{O} & \rightarrow \alpha + \alpha + \alpha + \alpha + n & Q = -14.437 \text{ MeV}
\end{align*}
\]

For further analysis, the condition that the calculated beam energy falls within 2 \(\sigma\) of the peak centroid was imposed, (lower limit of 34.53 MeV and an upper limit of 46.23 MeV). Selectively triggering on events within this peak ensures that most events correspond to the desired \(^9\)Be reaction, minimising the background signal.

5.4 Dalitz Plot Analysis

Further filtering techniques must be applied to the data in order to find out more about the excitation energy and breakup channels of the excited \(^9\)Be nuclei. A Dalitz plot is a scatterplot used to distinguish and gauge the relative frequency of various distinct ways in which products of (mainly 3-body) decays progress. Traditionally
5.4 Dalitz Plot Analysis

used in a particle physics context, they are used to distinguish the short-lived intermediate particle through which a decay progressed [16]. The $\alpha + ^9\text{Be}$ input channel leading to an excitation in $^9\text{Be}$ can result in several main output channels as detailed below [15].

\begin{align*}
^9\text{Be}^* & \rightarrow ^8\text{Be}^{gs} + n \rightarrow \alpha + \alpha + n \quad (20) \\
& \rightarrow ^8\text{Be}^{2+} + n \rightarrow \alpha + \alpha + n \quad (21) \\
& \rightarrow ^5\text{He}^{gs} + \alpha \rightarrow \alpha + \alpha + n \quad (22) \\
& \rightarrow \alpha + \alpha + n \quad (23)
\end{align*}

A Dalitz plot analysis of an event involves pairing up final state $\alpha$ particles and checking if they correspond to a decay of an intermediate state. Such an analysis can firstly allow the decay path of the excited state in $^9\text{Be}$ to be identified. The result of this is that several $^9\text{Be}$ excitation spectra can be produced, each specific to a particular decay path. Secondly, such a Dalitz plot analysis is crucial in determining which final state $\alpha$ particle is the scattered beam, and which particles arise from the decay of the excited target nucleus. This has been achieved in similar, previous work [17] and its importance is evident in section 2.3.

Since it is difficult to infer whether the $^9\text{Be}$ decayed via a direct breakup, two main decay pathways were explored during the Dalitz plot analysis; decays proceeding via states in $^8\text{Be}$ and those proceeding via the $^5\text{He}$ ground state. A typical event will result in the detection of three coincident $\alpha$ particles, as illustrated in figure 22. By calculating the angle of each of the detected particles in an event, the energies and momenta of these can be stored in an order that reflects their spatial distribution from right to left. The final distribution of particles depends on how the recoiling excited $^9\text{Be}$ decays. The beginnings of a simulation of this process was produced and is shown in Appendix E.

5.4.1 $^8\text{Be}$ Breakup Channel

Should the $^9\text{Be}$ decay proceed via a state in $^8\text{Be}$, the $^9\text{Be}$ decays via neutron emission. In this case, it is expected that the neutron and $^8\text{Be}$ daughter separate far more slowly than the velocity of the centre of mass of the system. An attempt to prove this using Matlab code is shown in Appendix E. The resulting $^8\text{Be}$ intermediate particle can be in one of several energy states. Upon decaying into two alpha particles, this energy is shared equally amongst the two decay products. It is expected that the two $\alpha$ particles resulting from this decay separate slowly and are are detected close together, having no spatial overlap with the scattered beam.
5.4 Dalitz Plot Analysis

Neighbouring alpha particles are therefore expected to originate from the same $^8$Be. Making reference to figure 22, particles $\alpha_1 + \alpha_2$ or $\alpha_2 + \alpha_3$ are likely to have resulted from a $^8$Be intermediate state, whereas $\alpha_1 + \alpha_3$ are not. This theory is confirmed by the Dalitz plot analysis shown shortly. If $\alpha_1 + \alpha_2$ resulted from the decay of an intermediate $^8$Be then $\alpha_3$ is the scattered projectile. Likewise if $\alpha_2 + \alpha_3$ originate from a decay of $^8$Be then $\alpha_1$ is the scattered beam particle.

![Diagram](image)

Figure 22: Diagram showing how the labelling of each final state $\alpha$ particle depends on its position of detection from right to left.

Within the sort code, for each event that passed the sum energy spectrum condition, the following formulae were used to calculate the decay Q-value of a theorised $^8$Be intermediate state corresponding to a pair of alpha particles, $\alpha_a$ and $\alpha_b$.

\[
\begin{align*}
Px/y/z(^8\text{Be}) &= (Px/y/z(\alpha_a) + Px/y/z(\alpha_b)) \\
|P(^8\text{Be})| &= \sqrt{Px(^8\text{Be})^2 + Py(^8\text{Be})^2 + Pz(^8\text{Be})^2} \\
Ek(^8\text{Be}) &= \frac{|P(^8\text{Be})|^2}{2m(^8\text{Be})}
\end{align*}
\]

From energy conservation:

\[
Q = E(\alpha_a) + E(\alpha_b) - Ek(^8\text{Be})
\]

For each set of paired particles, the calculated Q-value was added to a 1D histogram, producing a set of three Q-value spectra, shown in figures 23 and 24.
Figure 23: Q-value spectrum for proposed states in $^8$Be when reconstructed from alpha particles of closest separation, $\alpha_1$ and $\alpha_2$. An almost identical plot is generated when considering particles $\alpha_2$ and $\alpha_3$.

As can be seen, reconstructing the Q-value from pairs of neighbouring $\alpha$ particles results in a spectrum with peaks corresponding to known states in $^8$Be, consistent with spectra that have been reconstructed in similar previous work [15]. When reconstructing the Q-value for decays involving $\alpha$ particles of greatest separation, there is little evidence that they originate from states in $^8$Be as evidenced by the lack of known peaks in the
5.4 Dalitz Plot Analysis

Q-value spectrum.

To selectively gate on events that proceed via a particular state in $^8$Be, windows were placed on these peaks, and events that lay within $2\sigma$ of the peak were passed through for further analysis. To determine this, the spectrum shown in figure 23 was fit with a sum of Lorentzian curves as illustrated in figures 25 a) and 25 b). The fit parameters allowed the maximum and minimum values defining the window range for each peak to be defined as summarised in table 3.

![Graph 1](image1.png)

![Graph 2](image2.png)

Figure 25: a) Fit to the $^8$Be ground state peak. b) Fits to the 2+, 4+ and higher energy states in $^8$Be.

<table>
<thead>
<tr>
<th>State</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^8$Be 0+</td>
<td>0.1080</td>
<td>0.0806</td>
</tr>
<tr>
<td>$^8$Be Low 2+</td>
<td>0.7350</td>
<td>0.3874</td>
</tr>
<tr>
<td>$^8$Be 2+</td>
<td>3.840</td>
<td>1.307</td>
</tr>
</tbody>
</table>

Table 3: Details of the Q-value limits when gating on certain decay modes

An alternative way to visualize the data is by plotting the Q-values from different pairings onto 2D Dalitz plots as shown in figures 26 to 27.
5.4 Dalitz Plot Analysis

Figure 26: a) Dalitz plot indicating that particles 1:2, and 2:3 result from decays from states in $^8$Be. b) The regions of the plot windowed on when investigating certain decay modes. Note that the ground state window is too narrow to be seen on this plot.

Figure 27: Dalitz Plots when the other pairs of $\alpha$ particles are considered

By windowing on certain areas of the 2D plot shown in figure 26 b), with the limits defined in table 3, it
is possible to successfully identify both the decay mode of the event being investigated and which final state particle is the scattered $\alpha$ particle. With this being the case, an excitation energy in $^9$Be was calculated for each event, and raw excitation spectra corresponding to each decay mode were reconstructed.

### 5.4.2 $^5$He Decay Modes

To investigate the $^5$He breakup channel each of the final state $\alpha$ particles were paired with the inferred final state neutron. The following kinematics calculations were employed to calculate the Q-value for a decay from a possible state in $^5$He for each final state particle ($\alpha$):

\[
P_{x/y/z}(^5\text{He}) = (P_{x/y/z}(\alpha) + P_{x/y/z}(n))
\]  

(28)

\[
|P(^5\text{He})| = \sqrt{P_{x(^5\text{He})}^2 + P_{y(^5\text{He})}^2 + P_{z(^5\text{He})}^2}
\]  

(29)

\[
E_k(^5\text{He}) = \frac{|P(^5\text{He})|^2}{2m(^5\text{He})}
\]  

(30)

From energy conservation:

\[
Q = E(\alpha) + E(n) - E_k(^5\text{He})
\]  

(31)

This process was performed for each event read into the sort code that passed the sum energy spectrum filtering and three Q-value spectra were produced: one for each possible pairing of the neutron with a final state particle. The three spectra are shown in figure 28.
5.4 Dalitz Plot Analysis

Figure 28: Q-value spectra for proposed states in $^5$He when reconstructed from pairing each final state $\alpha$ particle with the inferred neutron. Additionally, these calculated Q-values for different $\alpha$ and neutron pairings were plotted on 2D Dalitz plots as shown in figures 29 to 31.

Figure 29: Dalitz plot of the Q-value when pairing $\alpha_1$ and the neutron vs Q-value when pairing $\alpha_2$ and the neutron and a proposed window.
The Dalitz plots show bands of high intensity at around 0.9 MeV consistent with the ground state of $^5$He. However, the bands appear to turn diagonal at higher energies, suggesting a background contribution from
breakup channels other than $^5\text{He}$. Similar plots have previously been produced in experiments where a $^6\text{Li}$ beam is used to excite the $^9\text{Be}$ target [15]. Since the scattered $^6\text{Li}$ beam particle could unambiguously be identified in the final state, a Dalitz plot analysis could be applied to the two final state $\alpha$ particles. This unambiguity means that the Dalitz plots are clearer and have less background. These results are shown in figure 32. These plots show the strength of the $^5\text{He}$ decay mode for various states in $^9\text{Be}$.

![Figure 32: Dalitz plot analysis for the $^5\text{He}$ decay mode of $^9\text{Be}$ when using inelastic $^6\text{Li}$ scattering [15]](image)

The $^5\text{He}$ breakup channel has not been examined further due to the time constraints of the project.

### 5.5 Excitation Calculations and Raw Spectra

Following the $^8\text{Be}$ Dalitz plot windowing process described in section 5.4.1, an excitation energy in $^9\text{Be}$ was calculated on an event by event basis. Histograms of these energies were plotted to form certain raw excitation spectra corresponding to different decay modes of $^9\text{Be}$. Three main decay modes were focussed on here. Decays proceeding via the $^8\text{Be}$ ground state, $^8\text{Be}$ 2+ state and the lower peak in the $^8\text{Be}$ 2+ state (primarily decays from the $^9\text{Be}$ 5/2+ state). These raw excitation spectra are shown next.
5.5 Excitation Calculations and Raw Spectra

Figure 33: Raw excitation spectrum corresponding to events where the $^9$Be decays via the $^8$Be ground state

Figure 34: Raw excitation spectrum corresponding to events where the $^9$Be decays via the $^8$Be 2+ state
5.5 Excitation Calculations and Raw Spectra

Figure 35: Raw excitation spectrum corresponding to events where the $^9$Be decays via the $^8$Be low 2+ peak in the Q-value spectrum.

Furthermore, a 2D plot of excitation energy versus total scattering angle of the scattered beam particle was drawn for the $^8$Be$^{\text{Low2}^+}$ breakup channel. This is shown in figure 36. The calculated excitation energy should not depend on scattering angle and so the fact that the line corresponding to the $^9$Be $5/2^+$ state is horizontal is a good indication that the detector position measurements are correct.
5.5 Excitation Calculations and Raw Spectra

5.5.1 Contaminant Background Suppression

As was seen in section 5.2, $^{12}$C and $^{16}$O contaminants are present in the target. Despite the event filtering methods described in sections 5.3 to 5.4, some events which involve a reaction with a contaminant will inevitably pass these filters and be plotted within the final excitation spectra. To overcome this, the data recorded for the $^{12}$C target were passed through the $^9$Be sort code and the resulting false excitation spectra corresponding to windowing on each $^8$Be breakup channel were recorded. These were subtracted from the $^9$Be spectra to correct for the contaminants. An example false excitation spectrum is shown in figure 37.
5.5 Excitation Calculations and Raw Spectra

Figure 37: Carbon background that passed the filtering methods for windowing on the $^8\text{Be}$ 2+ decay mode, before and after scaling. This spectrum is representative of the false events included in the excitation spectrum for $^9\text{Be}$, except on a much larger scale. This is due to the whole target being composed of $^{12}\text{C}$, rather than just contaminants. Therefore, these false spectra were scaled according to the relative amounts of $^{12}\text{C}$ in each target and the time of the beam run. This was determined by comparison of the raw energy versus total scattering angle data shown in figure 38.

![Graph showing excitation spectrum](image1)

Figure 38: Kinematic lines showing the relative concentrations of $^{12}\text{C}$ and $^{16}\text{O}$ in the $^9\text{Be}$ and $^{12}\text{C}$ targets respectively.
5.5 Excitation Calculations and Raw Spectra

A projection along the diagonal of these plots was made, which gave a 1D spectrum for each, with peaks that correspond to the projection of the $^{12}\text{C}$ and $^{16}\text{O}$ kinematic lines, as shown in figure 39. Buffit was then used to fit to the $^{12}\text{C}$ peaks in each and the total number of events within the peaks were recorded. For the $^{9}\text{Be}$ target, the number of $^{12}\text{C}$ ground state events was given as $(10,253.01 \pm 101.26)$. For the $^{12}\text{C}$ target, the number of events was $(123,722.4 \pm 351.75)$. Therefore a scaling factor of $(0.0828 \pm 0.00085)$ was applied to the $^{12}\text{C}$ false excitation spectra before subtraction from the $^{9}\text{Be}$ excitation spectra.

![Figure 39: 1D projection of the kinematic lines for $^{12}\text{C}$ and $^{16}\text{O}$ recorded for each target](image)

The corrected spectra and scaled backgrounds are shown in figures 40 to 43.

![Figure 40: Corrected excitation spectrum corresponding to events where the $^{9}\text{Be}$ decays via the $^{8}\text{Be}$ ground state](image)
5.5 Excitation Calculations and Raw Spectra

Figure 41: Corrected excitation spectrum corresponding to events where the $^9$Be decays via the $^8$Be 2+ state

Figure 42: Corrected excitation spectrum corresponding to events where the $^9$Be decays via the $^8$Be low 2+ peak in the Q-value spectrum
Figure 43: All corrected excitation spectra corresponding to each breakup path

The effects of the $^{16}$O contaminant were also overcome with this correction. It was seen that the $^{16}$O ground state kinematic line was approximately the same strength for both targets when factoring in the beam run time, most likely being due to a roughly similar level of oxidisation on the surface of each target. To fully eliminate the oxygen component though, a beam run with a purely $^{16}$O target must be recorded; however, this was not feasible with the current setup.

6 Secondary Data Analysis

The aim of the secondary analysis was to extract useful information regarding the energy states in $^9$Be from the measured excitation spectra. An approximate Matlab peak fitting method was employed to fit the known states in $^9$Be to the corrected excitation spectra that resulted from the previous analysis. The source code for this program is contained in Appendix G. This was performed for several main reasons. Firstly, directly fitting the resonances corresponding to known states with fixed widths and energies can confirm that the spectra measured during the beam run are consistent with previous studies. Secondly, in the case of a poor fit to the experimental data, the existence of new states can be theorised and introduced to the fitting routine.

The aim of segregating certain decay modes of $^9$Be and reconstructing excitation spectra for these modes individually was to study the relative branching ratios for decays between $^9$Be and excitations in the intermediate
6.1 Peak Fitting of the Excitation Spectra Description

The approximate peak fitting method is based on the Breit-Wigner distribution, which describes the full reaction cross section for an isolated resonance in an excitation spectrum with no scattering interference. In a simple case of the reaction \( a + b \rightarrow c + d, a(b,c)d \), a resonance at an energy \( E_R \) is described by:

\[
\sigma(E) = \frac{\pi}{k^2} g \frac{\Gamma_{ab} \Gamma_{cd}}{(E - E_R)^2 + \Gamma^2/4}
\]

(32)

Where \( \Gamma_{ab} \) and \( \Gamma_{cd} \) are the partial widths for the input and output channels. \( g \) is a factor that depends on the particle spins and \( I \) is the total angular momentum of the resonance [18].

\[
g = \frac{2I + 1}{(2s_a + 1)(2s_b + 1)}
\]

(33)

\[
I = s_a + s_b + L
\]

(34)

For the primary reaction under investigation, \(^9\text{Be}(\alpha, \alpha)^9\text{Be}^*\), this equation can be specialised depending on the breakup path of the excited \(^9\text{Be}^*\) state, which corresponds to different exit channels in the above Breit-Wigner equation. A simplification of the Breit-Wigner formula is achieved by introducing an amplitude constant, \( A \), leading to the following formulae for each breakup channel.

\[
^9\text{Be}^* \rightarrow ^8\text{Be}^{gs} + n \rightarrow \alpha + \alpha + n : \quad \sigma(E|E_R, \Gamma, A_{^8\text{Be}^{gs}}) = A_{^8\text{Be}^{gs}} \frac{\Gamma/2^2}{(E - E_R)^2 + \Gamma/2^2}
\]

(35)

\[
\rightarrow ^8\text{Be}^{2+} + n \rightarrow \alpha + \alpha + n : \quad \sigma(E|E_R, \Gamma, A_{^8\text{Be}^{2+}}) = A_{^8\text{Be}^{2+}} \frac{\Gamma/2^2}{(E - E_R)^2 + \Gamma/2^2}
\]

(36)

\[
\rightarrow ^5\text{He}^{gs} + \alpha \rightarrow \alpha + \alpha + n : \quad \sigma(E|E_R, \Gamma, A_{^5\text{He}^{gs}}) = A_{^5\text{He}^{gs}} \frac{\Gamma/2^2}{(E - E_R)^2 + \Gamma/2^2}
\]

(37)

\[
\rightarrow \alpha + \alpha + n : \quad \sigma(E|E_R, \Gamma, A_{\text{direct}}) = A_{\text{direct}} \frac{\Gamma/2^2}{(E - E_R)^2 + \Gamma/2^2}
\]

(38)

These simplified Lorentzian distributions provide the theoretical form for the peaks in each of the \(^9\text{Be}\) particle states. The areas of each peak in the individual \(^9\text{Be}\) excitation spectra allow the branching ratios for each breakup mode of the state to be determined. The branching ratios can be calculated on a theoretical level and comparisons between theory and experiment allow the spin and parity of the measured states to be determined.
6.1 Peak Fitting of the Excitation Spectra Description

excitation spectra. However, in reality, the measured spectra are subject to smearing of the peaks due to various sources of error in the experiment. Therefore, the widths of the measured peaks are broader than those predicted by the intrinsic widths of the states. This effect can be incorporated analytically by convolving the simplified Lorentzian distributions with a Gaussian. This convolution gives a Voigt Breit-Wigner distribution, the form of which is shown in equation 39.

\[
V_j(E|E_j, A_j, \Gamma_j, \sigma') = \frac{A_j \Gamma_j}{2 \sigma'} \sqrt{\frac{\pi}{2}} \text{Re} \left[ w \left( \frac{(E - E_j) + i \Gamma_j}{\sqrt{2 \sigma'}} \right) \right]
\]  

(39)

Where \( \text{Re}[w(z)] \) is the real part of the Faddeeva (complex complementary error) function which can be evaluated using standard Matlab functions [19]. A linear sum of such peaks are fitted to the data, which assumes constructive interference between states in the spectrum. In general, this will not be the case and so is purely an approximation of the true system. Therefore, the full function that is fit to the data is:

\[
F(E|E_j, A_j, \Gamma_j, \sigma') = \sum_j V_j(E|E_j, A_j, \Gamma_j, \sigma')
\]

(40)

\[
= \sum_j \frac{A_j \Gamma_j}{2 \sigma'} \sqrt{\frac{\pi}{2}} \text{Re} \left[ w \left( \frac{(E - E_j) + i \Gamma_j}{\sqrt{2 \sigma'}} \right) \right]
\]

(41)

The effect of the experimental resolution, \( \sigma' \), on the appearance of the Voigt distribution can be seen for a constant Lorentzian width \( \Gamma (\gamma) \) in figure 44 [20].

![Figure 44: Plots showing the effect of increasing experimental error on the appearance of a peak of intrinsic width \( \gamma \), in the measured excitation spectrum](image)

Figure 44: Plots showing the effect of increasing experimental error on the appearance of a peak of intrinsic width \( \gamma \), in the measured excitation spectrum
6.2 Determination of the Experimental Resolution

The Matlab peak fitting program requires that upper and lower limits are placed on the parameters that define the function shown in equation 41. When fitting the known states of $^9$Be to the excitation spectra, online databases [21] were used to constrain the energies and widths of each known state, with the upper and lower limits being defined only by the experimental errors obtained by the previous studies. Therefore, for most peaks in the spectrum, $E_j$ and $\Gamma_j$ are known to good precision. The amplitude, $A_j$, of each peak is given a broad range of values that the fit can take. However, to implement the program, a value for the experimental resolution $\sigma'$ is required. The determination of this is the focus of the following section.

The Matlab fitting function used does not provide standard errors on the parameters optimised during the fit. As will be seen in section 6.5.2, the lack of errors does make it hard to fully interpret the final results. Programming this into the fitting routine is the next logical step for development.

6.2 Determination of the Experimental Resolution

Despite the fact that the sum energy spectrum and Dalitz plot filtering methods were applied to the data, requiring that the excited $^9$Be underwent a breakup via a state in $^8$Be, a small peak at an excitation energy of 0 MeV was seen in each of the three spectra, corresponding to the $^9$Be ground state. This demonstrates the imperfections of these filtering techniques. Since the ground state is stable, its intrinsic width is infinitely thin. Therefore, the width of this peak in the measured spectra is purely due to the gaussian experimental resolution. A gaussian fit to this peak was made, as shown in figure 45. This gave the following resolution that could be used within the peak fitting program as a common parameter amongst all peaks:

\[
FWMH = (0.638 \pm 0.066) MeV \quad (42)
\]

\[
\sigma' = (0.272 \pm 0.028) MeV \quad (43)
\]
6.3 Fit to the Known States in $^9$Be

Fitting the currently known states of $^9$Be to each of the excitation spectra resulted in the following fits.

Figure 45: Fit to the ground state peak in the 2+ windowed $^9$Be excitation spectrum

Figure 46: Diagram showing the peak fitting to the spectrum corresponding to decays via the ground state in $^8$Be
6.3 Fit to the Known States in $^9\text{Be}$

Figure 47: Diagram showing the peak fitting to the spectrum corresponding to decays via the ground 2+ state in $^8\text{Be}$

Figure 48: Diagram showing the peak fitting to the spectrum corresponding to decays via the lower end of the 2+ state in $^8\text{Be}$

The fit is consistently poor between 8-13 MeV, suggesting the possibility of new physics in this region. The curve generally fits well to the data at low and high energies. It was expected that windowing on the low 2+ peak in the $^8\text{Be}$ Q-value Dalitz plots would result in only the $5/2^+$ state being seen in the spectrum. Figure 48 clearly shows that this is not fully the case, again highlighting the current failings of the Dalitz plot windowing process.
6.4 Introduction of New States to the Fitting Routine

In order to test the hypothesis that new states reside within the region of poor fit, new states were introduced into the fitting routine with very broad limits on their centroids, widths and amplitudes. Two possible situations were investigated:

1. Two new states introduced with centroids between 8.5-10.5 MeV and 12-13.5 MeV and both with widths between 0.1-4 MeV

2. A single new state introduced with a centroid in the region of 8-13 MeV and a width between 0.1-5 MeV

6.4.1 Introduction of a Single New State

The introduction of a single new peak into the fitting routine results in the following fits for the spectra, corresponding to decays via the $^8$Be ground state and 2+ state respectively. When calculating the value of $\chi^2$ for each fit, the error on each point was taken to be the square root of the data value at that point.

![Figure 49: Diagram showing the peak fitting, including the new peak, to the spectrum corresponding to decays via the ground state in $^8$Be](image)

The fit statistics for the new peak and two existing states within this region are shown in table 4. The reduced $\chi^2$ value for this fit is 0.5526.
6.4 Introduction of New States to the Fitting Routine

<table>
<thead>
<tr>
<th>State</th>
<th>Energy (MeV)</th>
<th>Width (MeV)</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>New State</td>
<td>11.0564</td>
<td>5.0000</td>
<td>265.7677</td>
</tr>
<tr>
<td>Existing State ≈ 11.2 MeV</td>
<td>11.3070</td>
<td>0.5255</td>
<td>0.4443</td>
</tr>
<tr>
<td>Existing State ≈ 11.81 MeV</td>
<td>11.8300</td>
<td>0.3714</td>
<td>5.2107</td>
</tr>
</tbody>
</table>

Table 4: Fit statistics when introducing the existence of a new state to the fitting routine, when fitting the spectrum corresponding to the $^8$Be ground state breakup mode

Figure 50: Diagram showing the peak fitting, including the new peak, to the spectrum corresponding to decays via the ground 2+ state in $^8$Be

The fit statistics for the new peak and two existing states within this region are shown in table 5. The reduced $\chi^2$ value for this fit is 1.2409.

<table>
<thead>
<tr>
<th>State</th>
<th>Energy (MeV)</th>
<th>Width (MeV)</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>New State</td>
<td>11.1943</td>
<td>4.9690</td>
<td>565.3937</td>
</tr>
<tr>
<td>Existing State ≈ 11.2 MeV</td>
<td>11.3069</td>
<td>0.5250</td>
<td>0.1074</td>
</tr>
<tr>
<td>Existing State ≈ 11.81 MeV</td>
<td>11.8300</td>
<td>0.4129</td>
<td>15.6880</td>
</tr>
</tbody>
</table>

Table 5: Fit statistics when introducing the existence of a new state to the fitting routine, when fitting the spectrum corresponding to the $^8$Be 2+ breakup mode

Clearly the introduction of this single peak does not greatly improve the fit. As can be seen, there is still a noticeable discrepancy between the fit and the data within the region of 8-13 MeV. Furthermore, the width of this state that was optimised during the fit is so large that the tails extend beyond the bounds of the measured spectrum, distorting the originally good fit at high and low energies. The new introduced state is by far the
most dominant peak in the fit and greatly suppresses the other states within the region of interest. It seems unlikely that the state should have this level of dominance in the fit, further suggesting that the fitting of a single peak is not the correct approach. This approach is not continued in further analysis.

### 6.4.2 Introduction of Two New States

The introduction of two new peaks into the fitting routine results in the following fits for the spectra corresponding to decays via the $^8$Be ground state and 2+ state respectively.

![Diagram showing the peak fitting, including the two new peaks, to the spectrum corresponding to decays via the $^8$Be ground state in $^8$Be](image)

The fit statistics for the two new peaks and two existing states within this region are shown in table 6. The reduced $\chi^2$ value for this fit is 0.3421.

<table>
<thead>
<tr>
<th>State</th>
<th>Energy (MeV)</th>
<th>Width (MeV)</th>
<th>Amplitude</th>
<th>Area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>New State 1</td>
<td>9.9614</td>
<td>3.4392</td>
<td>223.2612</td>
<td>1138.8</td>
</tr>
<tr>
<td>New State 2</td>
<td>12.7488</td>
<td>1.2499</td>
<td>171.7535</td>
<td>331.7788</td>
</tr>
<tr>
<td>Existing State</td>
<td>11.26788</td>
<td>0.6188</td>
<td>81.9286</td>
<td>78.9207</td>
</tr>
<tr>
<td>Existing State</td>
<td>11.81 MeV</td>
<td>0.4119</td>
<td>78.5122</td>
<td>50.5060</td>
</tr>
</tbody>
</table>

Table 6: Fit statistics when introducing the existence of two new states to the fitting routine, when fitting the spectrum corresponding to the $^8$Be ground state breakup mode.
6.4 Introduction of New States to the Fitting Routine

Figure 52: Diagram showing the peak fitting, including the two new peaks, to the spectrum corresponding to decays via the 2+ state in $^8$Be

The fit statistics for the two new peaks and two existing states within this region are shown in table 7. The reduced $\chi^2$ value for this fit is 0.5772.

<table>
<thead>
<tr>
<th>State</th>
<th>Energy (MeV)</th>
<th>Width (MeV)</th>
<th>Amplitude</th>
<th>Area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>New State 1</td>
<td>9.7417</td>
<td>2.6410</td>
<td>473.5145</td>
<td>1878</td>
</tr>
<tr>
<td>New State 2</td>
<td>12.7915</td>
<td>1.6293</td>
<td>435.4091</td>
<td>1091</td>
</tr>
<tr>
<td>Existing State</td>
<td>$\approx$ 11.2 MeV</td>
<td>0.6250</td>
<td>256.8621</td>
<td>249.8797</td>
</tr>
<tr>
<td>Existing State</td>
<td>$\approx$ 11.81 MeV</td>
<td>0.3704</td>
<td>212.8586</td>
<td>123.2197</td>
</tr>
</tbody>
</table>

Table 7: Fit statistics when introducing the existence of two new states to the fitting routine, when fitting the spectrum corresponding to the $^8$Be 2+ breakup mode

The introduction of the two new states provides a better fit to the experimentally measured spectra, especially in the region between 8-13 MeV, where the fit of all known states was very poor. What is particularly striking about both of these fits is that despite the large range of values that the energies and widths of these states could have taken, the optimised values of these parameters are fairly consistent between each of the fits, as summarised in the following table.
Table 8: A comparison of the fit parameters of the two new states between each of the spectra

<table>
<thead>
<tr>
<th>State</th>
<th>Decay Mode Spectrum</th>
<th>Energy (MeV)</th>
<th>Width (MeV)</th>
<th>Amplitude</th>
<th>Area (counts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>New State 1</td>
<td>³Be GS</td>
<td>9.9614</td>
<td>3.4392</td>
<td>223.2612</td>
<td>1138.8</td>
</tr>
<tr>
<td>New State 1</td>
<td>³Be 2+</td>
<td>9.7417</td>
<td>2.6410</td>
<td>473.5145</td>
<td>1878</td>
</tr>
<tr>
<td>New State 2</td>
<td>³Be GS</td>
<td>12.7488</td>
<td>1.2499</td>
<td>171.7535</td>
<td>331.7788</td>
</tr>
<tr>
<td>New State 2</td>
<td>³Be 2+</td>
<td>12.7915</td>
<td>1.6293</td>
<td>435.4091</td>
<td>1091</td>
</tr>
</tbody>
</table>

The area of the peaks within each of the fits allows the relative branching ratios for the breakup of each measured state in ⁹Be to be determined. The relative branching ratio for the breakup of a particular ⁹Be state to the ³Be⁰s or ³Be²⁺ is defined as:

\[
BR = \frac{\text{Peak Area}(³\text{Be}^{⁰s} \text{ spectrum})}{\text{Peak Area}(³\text{Be}^{²⁺} \text{ spectrum})}
\]  

(44)

For the four states in question, the measured relative branching ratios are summarised in the following table.

Table 9: A comparison of the measured relative branching ratios for breakups into each ³Be state

<table>
<thead>
<tr>
<th>³Be State</th>
<th>Relative BR</th>
</tr>
</thead>
<tbody>
<tr>
<td>New State 1</td>
<td>0.6064</td>
</tr>
<tr>
<td>New State 2</td>
<td>0.3041</td>
</tr>
<tr>
<td>Existing State ≈ 11.2 MeV</td>
<td>0.3158</td>
</tr>
<tr>
<td>Existing State ≈ 11.81 MeV</td>
<td>0.4099</td>
</tr>
</tbody>
</table>

6.5 Centrifugal Barrier Penetrability Calculations

The spins and parities of the states under investigation can be determined by comparing their experimentally measured branching ratios with those theoretically predicted by centrifugal barrier penetrability calculations.

6.5.1 Theory of Decay Rates

⁹Be decays to a ³Be daughter through the emission of a neutron as shown in figure 53 a). Since the neutron has no electric charge, it is not required to penetrate a coulomb barrier. Rather, the only suppression of the decay originates from the centrifugal barrier which has the following functional form [22] and is plotted in figure 53 b).

\[
V_{\text{cent}}(r) = \frac{\ell(\ell + 1)\hbar^2}{2mr^2}
\]  

(45)
6.5 Centrifugal Barrier Penetrability Calculations

Where $\ell$ is the orbital angular momentum required to be carried away by the neutron during the decay. In a transition between an initial state and a final state with total angular momenta $J_i$ and $J_f$ respectively, the total angular momentum taken by the neutron can range between $|J_i + J_f|$ and $|J_i - J_f|$ [23]. The fact that a neutron has a spin and parity of $1/2^+$ constrains the values of the orbital angular momentum, $\ell$, required to be carried away by the neutron in a decay. Different spins and parities of the initial and final states therefore result in different centrifugal barriers that the emitted neutron must tunnel through.

![Figure 53: a) The emission of a neutron and angular momentum conservation in $^9$Be decay b) The functional form of the centrifugal barrier potential](image)

The decay rate for neutron emission is proportional to the centrifugal barrier penetrability and so is dependent on the spins and parities of the initial and final states along with the available phase space of the final state particles. The peak fitting described in section 6.4.2 provided the energies of the four measured states of interest. By using these values, several spins of the decaying state can be trialled and a decay rate to each of $^8$Be$^{9s}$ and $^8$Be$^{2+}$ can be calculated. This allows a theoretical branching ratio to be derived for each trialled spin of the initial state, which can be compared to the experimentally measured relative branching ratio. These calculations were performed using the ckin kinematics code by Dr Carl Wheldon.

6.5.2 Comparison with Experimental Results

The tables showing the calculated penetrabilities for various possible spins for the states in $^9$Be are included in Appendix H. A summary of the comparison and closest fits to the experimental results are included in table 10 below. The interpretation of these results is discussed in the conclusion section next.
Through inelastic $\alpha$ particle scattering, two excitation spectra for $^9\text{Be}$ were reconstructed. Each was specific to events corresponding to a breakup via the $^8\text{Be}$ ground state or $^8\text{Be}$ $2^+$ state. A consistently poor fit of the known states in $^9\text{Be}$ to the spectra in the region of 8-13 MeV suggested the possible existence of new states in this region.

The existence of a single new state when introduced to the fitting routine did not improve the fits in this region, and also distorted the fits at higher and lower energies. The introduction of two new states within the spectra vastly improved the fit in this region but did give suspiciously low values for the reduced $\chi^2$, suggesting that the introduction of these states may be an oversimplification of the true, underlying physics. However, the fit parameters for the two new states optimised by the fitting routine were remarkably consistent between the two spectra as shown in table 11, indicating that the introduction of these states was the correct approach.

The peaks within the spectra were analysed individually, with the area beneath each being used to gauge the relative branching ratio for $^9\text{Be}$ breakup into each state in $^8\text{Be}$. These were compared with those theoretically predicted by centrifugal barrier penetrability calculations. In addition to the two new states, existing states at 11.2 and 11.8 MeV were examined in this way. The known state at 11.8 MeV was found to have a spin of $5/2^{-}$ or $7/2^{-}$, which is consistent with the $5/2^{-}$ spin and parity known from previous studies. The state at 11.2 MeV was also found to have a spin of $7/2^{+}$ or $9/2^{+}$, which is consistent with it’s current assignment to the $1/2^{+}$ rotational band. Despite this agreement, the lack of standard errors on the parameters in the fitting routine means that the error on the area of each peak and hence also on the experimental branching ratios are
unknown. If sufficiently large, the errors on the branching ratio could encompass the theoretical predictions corresponding to several other spin states.

The experimental branching ratios of the two new states at $\approx 9.8$ and $12.8$ MeV gave the closest fit to the theoretical predictions corresponding to spin states of $(3/2^+ \text{ or } 5/2^+)$ and $(7/2^+ \text{ or } 9/2^+)$ respectively. Again, depending on the size of the errors on the experimental branching ratios, which are not yet known, the experimentally measured values could potentially include several other spin states as shown in Appendix H.

This preliminary analysis of the spins does not suggest that any of the states investigated can be assigned to the $3/2^-$ rotational band. However, one would expect the $\alpha$ particle scattering reaction to be particularly efficient at exciting $^9$Be to states with the same overall structure as the ground state (ones belonging to the ground state $3/2^-$ rotational band). As shown in figures 51 and 52, the two new states are very dominant in both spectra, suggesting that one of these could belong to the $3/2^-$ band.

8 Future Improvements

Several improvements can be made in the near future that may shed further light on the nature of the states in question. As has been mentioned, an implementation of a peak fitting routine that provides standard errors on the fitted parameters will allow for less ambiguity in the labelling of the spins of the states. $^9$Be decays proceeding via the $4^+$ state in $^8$Be were not investigated due to a lack of statistics, though this could easily be solved by undertaking a longer beam run. Creating a spectrum corresponding to breakups via this state would provide a further set of branching ratios that could be compared with theory for further verification of the findings. An investigation into the $^5$He breakup channel would also contribute to this, though it is difficult to see how this channel could be identified when examining this reaction with three final state $\alpha$ particles.

Additionally, a thorough data analysis would fully factor the difficulties of focussing on a single breakup channel within the Dalitz plots. Figure 25 shows that selectively gating on peaks corresponding to a breakup via a particular state in $^8$Be will still include events from the tail of another state. To focus on a single breakup channel alone, the contributions from other channels must be removed. To quantify this contribution would require a Monte Carlo analysis of the system. A factoring of the spatial variation of the detection efficiency is also required for a full analysis which could be achieved through a more advanced model similar to that shown in Appendix E.
Finally, the experimental contribution to the widths of the peaks in the excitation spectra is around 600 KeV, which is far greater than the detector resolution discussed in section 3.6. Therefore, the error mainly originates from other experimental factors such as the calculations of the angles of the detected particles. This could be overcome by moving the detectors further from the source, to minimise the solid angle subtended by each detector strip. Accurate measurements of the distances and angles from the target to the detectors will also improve this resolution.

9 Acknowledgements

I would like to firstly thank my project supervisor, for his continual help and guidance throughout this project. The time that he has regularly devoted to explaining the theoretical background, checking our analysis methods and providing useful feedback to assessed work has been much appreciated. I would also like to extend this thanks to other members of the nuclear physics group including research students who have helped us along the way. Furthermore, I wish to thank the other students undertaking the project for their hard work. Especially to Jacob Hall for putting up with me for this long. Finally, I must wholeheartedly thank my girlfriend. Because of her understanding, unlike the $^9$Be states being measured on the cyclotron, we did not 'break up.'
Appendices

A  Deformed Harmonic Oscillator Model

Figure 54: Plot of the energy levels of the deformed harmonic oscillator (in units of $\hbar\omega$) as a function of deformation eccentricity, $\epsilon$

B  Detector Resolution Calculations

B.1  Calculation of the Intrinsic Detector Resolution from Alpha Particle Spectra: Matlab Code

```matlab
clear all;
close all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Centroids and FWHM of the Peaks
% And the energy of the 3 alpha particles
%Along with errors

x = [5.143 5.49 5.77];
B.1 Calculation of the Intrinsic Detector Resolution from Alpha Particle Spectra: Matlab Code

```matlab
12 y = [449.63 483.61 515.76];
13 FWHM = [6.98 6.64 7.06];
14 eFWHM = [0.04 0.04 0.09];
15 e = [0.02 0.02 0.06];
16 w = 1./(times(e,e));
17
18 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
19
20 \% Energy Calibration
21
22 figure
23 errorbar(x,y,e,’.k’)
24 hold
25 xlabel(‘Energy (MeV)’)
26 ylabel(‘Peak Centroid (Channels)’)
27
28 \% Linear fit
29
30 n=1;
31 [p,s] = polyfit(x,y,n);
32 ste = sqrt(diag(inv(s.R)*inv(s.R’)))*s.normr.^2/s.df);
33
34 \% Calculate FWHM and energies along with errors in units of MeV
35
36 E = (y–p(2))/p(1);
37
38 Eerr = sqrt((ste(1)+(y–p(2))/p(1).^2)+ (e*p(2)/p(1)).^2);
39
40 plot[E,y];
41
42 FWHM_E = FWHM/p(1);
43
44 FWHM_Eerr = sqrt((FWHM*ste(1)/(p(1).^2)+(eFWHM/p(1)).^2));
45
46 \% Plot of absolute resolution
47
48 figure
49 errorbar(E,FWHM_E,FWHM_Eerr,’.k’)
50 xlabel(‘E (MeV)’)
51 ylabel(‘Resolution (MeV)’)
```
% Fractional Resolution

Res = FWHM_E./E;

Res_err = sqrt( (FWHM_Eerr./E).^2 + (Eerr.*FWHM_E./(E.^2)).^2 );

figure
errorbar(E,Res,Res_err,'.k')
xlabel('E (MeV)')
ylabel('Fractional Resolution')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate fit parameters

Log_E= log10(E);
Log_Res= log10(Res);
Log_err= Res_err./Res;
w1 = 1./times(Log_err,Log_err);

figure
errorbar(Log_E,Log_Res,Log_err,'.k')
xlabel('Log(E (MeV))')
ylabel('Log(Fractional Resolution)')
hold

n=1;
[p2,s2] = polyfit(Log_E,Log_Res,n);
ste = sqrt(diag(inv(s2.R)*inv(s2.R'))).*s2.normr.^2./s2.df);

Y2=p2(1)*Log_E + p2(2);
plot(Log_E,Y2);

Power = p2(1)
Error_Constant = ste(1)
Constant = p2(2)
Error_CONSTANT = ste(2)
B.2 Calculation of the Average Intrinsic Detector Resolution: Fortran Code

```fortran
1 c Process all the detectors
2
3 np=0
4 np1=0
5 np2=0
6 np3=0
7 np4=0
8
9 if(n1f.ge.1) then
10   do i=1,min(n1f,n1b)
11     np=np+1
12     np1=np1+1
13     thy1(i) = atan(x1(i)/r1x(i))
14     e(np)=e1f(i)
15     eb(np)=e1b(i)
16     px(np)=sqrt(2*e(np))*sin(thx1(i))*cos(thy1(i))
17     py(np)=sqrt(2*e(np))*sin(thy1(i))
18     pz(np)=sqrt(2*e(np))*cos(thx1(i))*cos(thy1(i))
19     th_tot(np)=acos(cos(thx1(i))*cos(thy1(i)))*57.2*thx1(i)/abs(thx1(i))
20   enddo
21 endif
22
23 if(n2f.ge.1) then
24   do i=1,min(n2f,n2b)
25     np=np+1
26     np2=np2+1
27     thy2(i) = atan(x2(i)/r2x(i))
28     e(np)=e2f(i)
29     eb(np)=e2b(i)
30     px(np)=sqrt(2*e(np))*sin(thx2(i))*cos(thy2(i))
31     py(np)=sqrt(2*e(np))*sin(thy2(i))
32     pz(np)=sqrt(2*e(np))*cos(thx2(i))*cos(thy2(i))
33     th_tot(np)=acos(cos(thx2(i))*cos(thy2(i)))*57.2*thx2(i)/abs(thx2(i))
34   enddo
35 endif
36
37 if(n3f.ge.1) then
38   do i=1,min(n3f,n3b)
39     np=np+1
40     np3=np3+1
41   enddo
42 endif
```

B.3 Calculation of the Intrinsic Detector Resolution Over a Large Energy Range: Matlab Code

```matlab
thy3(i) = atan(x3(i)/r3x(i))
e(np)=e3f(i)
eb(np)=e3b(i)
px(np)=sqrt(2+4.*e(np))*(sin(thx3(i))*cos(thy3(i)))
py(np)=sqrt(2+4.*e(np))*sin(thy3(i))
pz(np)=sqrt(2+4.*e(np))*cos(thx3(i))*cos(thy3(i))
th_tot(np)=acos(cos(thx3(i))*cos(thy3(i)))+57.2*thx3(i)/abs(thx3(i))

enddo
endif
endif
if(n4f.ge.1) then
  do i=1,min(n4f,n4b)
    np=np+1
    np4=np4+1
    thy4(i) = atan(x4(i)/r4x(i))
e(np)=e4f(i)
eb(np)=e4b(i)
px(np)=sqrt(2+4.*e(np))*(sin(thx4(i))*cos(thy4(i)))
py(np)=sqrt(2+4.*e(np))*sin(thy4(i))
pz(np)=sqrt(2+4.*e(np))*cos(thx4(i))*cos(thy4(i))

  th_tot(np)=acos(cos(thx4(i))*cos(thy4(i)))+57.2*thx4(i)/abs(thx4(i))
  enddo
  endif
endif

c=---------------------------------------------------------------
c Resolution Measurements
c Plot energy difference between the front and back detectors against the
c average detected energy
do i=1,np
call inc2d(21,nint(((e(i)+eb(i))*7/2)),nint(((e(i)-eb(i))*100)+128))
endo
```

B.3 Calculation of the Intrinsic Detector Resolution Over a Large Energy Range: Matlab Code
% Centroids and FWHM of the Peaks
% from the 1D cuts of the 2D delta E vs E plot
% Along with errors

E = [2.714 4.143 5.571 7.957 12.714 15.571];
FWHM_old = [10.36 8.41 7.85 8.40 10.49 12.66 14.89];
FWHM = FWHM_old/100;
eFWHM_old = [0.02 0.02 0.02 0.02 0.03 0.05 0.07];
eFWHM = eFWHM_old/100;

% Plot absolute resolution vs Energy

figure
errorbar(E,FWHM,eFWHM,'.k')
hold
xlabel('Energy (MeV)','FontSize',14)
ylabel('FWHM (MeV)','FontSize',14)
title('Absolute Resolution','FontSize',14)

% Fractional Resolution

Res = FWHM./E;
Res_err = sqrt((eFWHM./E).^2);

figure
errorbar(E,Res,Res_err,'.k')
xlabel('E (MeV)','FontSize',14)
ylabel('Fractional Resolution','FontSize',14)
title('Fractional Resolution','FontSize',14)

% Calculate fit parameters

Log_E = log10(E);
B.3 Calculation of the Intrinsic Detector Resolution Over a Large Energy Range: Matlab Code

```matlab
46 Log_Res = \log_{10}(Res); 
47 Log_err = Res / Res; 
48 w1 = 1./times(Log_err,Log_err)); 
49 
50 figure 
51 errorbar(Log_E,Log_Res,Log_err,'.k') 
52 xlabel('Log(E (MeV))','FontSize',14) 
53 ylabel('Log(Fractional Resolution)','FontSize',14) 
54 title('Log Fractional Resolution Fit','FontSize',14) 
55 hold 
56 
57 n=1; 
58 [p2,s2] = polyfit(Log_E,Log_Res,n); 
59 ste = sqrt(diag(inv(s2.R)*inv(s2.R'))).*s2.normr.^2./s2.df; 
60 
61 Y2=p2(1)*Log_E + p2(2); 
62 
63 plot(Log_E,Y2); 
64 
65 Power = p2(1) 
66 Error_Constant = ste(1) 
67 
68 Constant = p2(2) 
69 Error_Constant = ste(2) 
70 
71 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
72 
73 % Plot the fit 
74 
75 figure 
76 errorbar(E,Res,Res_err,'.k') 
77 xlabel('E (MeV)','FontSize',14) 
78 ylabel('Fractional Resolution','FontSize',14) 
79 title('Fractional Resolution Fit','FontSize',14) 
80 hold 
81 
82 Y3 = 0.057*E.^(-0.7487) 
83 
84 plot(E,Y3) 
```
C Calculation of the Angle of each Detected Particle

\[ \theta, \phi, d \text{ and } x \text{ are known. The total detection angle is } \theta_x = \theta + \alpha. \text{ From trigonometry:} \]

\[
\alpha = \arcsin\left(\frac{x \sin(\phi)}{r}\right) \quad (46)
\]

\[
r = \sqrt{d^2 + x^2 - 2dx \cos(\phi)} \quad (47)
\]

Likewise, the downwards angle \( \theta_y \) is given by:

\[
\theta_y = \arctan\left(\frac{y}{r}\right) \quad (48)
\]

where \( y \) is the downwards length analogous to \( x \). From these two angles the total angle can be calculated \( \theta_{tot} \).

From the detected energy of a particle, it’s assumed mass of \( \approx 4 \text{ AMU} \) and these angles, the \( x, y \) and \( z \) momenta of the particle can be calculated.
D Derivation of the Theoretical Kinematic Lines

The kinematic lines are calculated in the following section of the sort code:

```plaintext
1 c Plotting the theorised scattered beam energy vs scattering angle
2
3 Ma=4.00151
4 Mr=9.0122
5 thDeg=-110
6 c Uncomment the do loop and plotting functions to plot the lines
7 c do thDeg=0,100
8
```
Mr=9.0122

thRad=thDeg/57.3
Vb=sqrt(2*Ma*Ebeam)/Ma
Vcm=Vb*(Ma/(Ma+Mr))
Vbcm=sqrt(2*Ma*Ebeam)*(Mr/(Ma*(Ma+Mr)))
EBCM=0.5*Ma*(Vbcm**2)
Etm=0.5*Mr*(Vcm**2)

Ecm=Ebcm+Etcm
Eacm=Ecm*(Mr/(Ma+Mr))
Ercm=Ecm*(Ma/(Ma+Mr))
Vacm=sqrt(2*Ma*Eacm)/Ma
VaLab=sqrt(Vcm**2 + Vacm**2 - 2*Vcm*Vacm*cos(3.142 - abs(thRad)))
EaLab=0.5*Ma*(VaLab**2)
thRaDLab=asin(Vacm*sin(thRad)/VaLab)
thDegLab=thRaDLab*57.3
c call inc2d(3, nint(thDegLab+128), nint(EaLab+5))
\[ V_b = \sqrt{2 \cdot M_a \cdot E_{beam}} / M_a \]
\[ V_{cm} = V_b \cdot (M_a / (M_a + M_r)) \]
\[ V_{bcm} = \sqrt{2 \cdot M_a \cdot E_{beam}} \cdot (M_r / (M_a \cdot (M_a + M_r))) \]
\[ E_{bc} = 0.5 \cdot M_a \cdot (V_{bcm} \cdot 2) \]
\[ E_{cm} = 0.5 \cdot M_r \cdot (V_{cm} \cdot 2) \]
\[ E_{tcm} = 0.5 \cdot (V_{bc} + E_{bc}^2) \]
\[ E_{ac} = E_{cm} \cdot (M_r / (M_a + M_r)) \]
\[ E_{rc} = E_{cm} \cdot (M_a / (M_a + M_r)) \]
\[ E_{a} = \sqrt{2 \cdot M_a \cdot E_{ac}} / M_a \]
\[ V_{a} = \sqrt{V_{cm} + 2 \cdot V_{bc} + V_{bc} \cdot \cos(3.142 \cdot \text{abs}(\text{thRad})}) \]
\[ E_{a} = 0.5 \cdot M_a \cdot (V_{a} \cdot 2) \]
\[ \text{th}_{\text{ra}} = \text{asin}(V_{ac} \cdot \sin(\text{thRad}) / V_{a}) \]
\[ \text{th}_{\text{deg}} = \text{th}_{\text{ra}} \cdot 57.3 \]
\[ \text{c call inc2d}(3, \text{nint}(\text{th}_{\text{deg}} + 128), \text{nint}(E_{a} \cdot 5)) \]

E Event Simulations

The following code attempts to calculate the distribution of final state particles across the detector array, in a simplified 1D system. The aim was to determine the spatial separation of the two \( \alpha \) particles resulting from breakup and to check the frequency with which they overlap with the scattered beam.
Tha1=zeros(1,140);
Ea2=zeros(1,140);
Tha2=zeros(1,140);

figure
hold

for thDeg=1:36,

%%%%%%

% Calculate the direction of the recoiling 9Be *

Ex9Be=1.67;

thDeg1=(thDeg-18)*5;

thRad=thDeg1/57.3;
Vb=sqrt((2*Ma*Ebeam)/Ma);
Vcm=Vb*(Ma/(Ma+Mr));
Vbcm=sqrt((2*Ma*Ebeam)*(Mr/(Ma*(Ma+Mr))));
Ebcm=0.5*Ma*(Vbcm^2);
Etcm=0.5*Mr*(Vcm^2);
Ecm=Ebcm+Etcm-Ex9Be;
Eacm=Ecm+(Mr/(Ma+Mr));
Ercm=Ecm+(Ma/(Ma+Mr));

%%%%%%

% Calc energy and angle of scattered alpha

Vacm=sqrt((2*Ma*Eacm)/Ma);
VaLab=sqrt(Vcm^2 + Vacm^2 -2*Vcm*Vacm*cos(thRad));
EaLab=0.5*Ma*(VaLab^2);
thRaDLab=asin(Vacm*sin(thRad)/VaLab);
thDegLab=thRaDLab*57.3;

%%%%%%

% Calc energy and angle of recoiling 9Be

Vrcm=sqrt((2*Mr*Ercm)/Mr);
VrLab=sqrt(Vcm^2 + Vrcm^2 -2*Vcm*Vrcm*cos(3.142-thRad));
ErLab=0.5*Mr*(VrLab^2);
thRaDLab=asin(Vrcm*sin(3.142-thRad)/VrLab);
thDegLab=thRaDLab*57.3;
\text{thRad1tot} = \text{thRaDLab};
\text{thDeg1tot} = \text{thDegLab}

\text{MSc} = 9817614;

\text{Vx} = \text{VrLab} \times \sin(\text{thRaDLab});
\text{Vz} = \text{VrLab} \times \cos(\text{thRaDLab});

\text{Vt1} = \sqrt{(\text{Vx}^2 + \text{Vz}^2)};
\text{Et1} = 0.5 \times \text{Mr} \times \text{Vt1}^2;

\text{thxDeg1} = \text{thDegLab};
\text{E(\text{thDeg}) = ErLab};
\text{Th1(\text{thDeg}) = thDeg1tot};

\text{%scatter(thDeg1tot, ErLab)};

\text{for k=1:180,}

\text{th2Deg} = (k - 90);
\text{th2Rad} = \text{th2Deg} / 57.3;

\text{Mn} = 1;
\text{Mr2} = 8;
\text{Ex8Be} = 3.03;
\text{Q1} = -1.57;
\text{Vcm} = \text{Vt1};
\text{Ecm} = \text{Et1} + \text{Q1} \times \text{Ex8Be} - \text{Ex8Be};
\text{Encm} = \text{Ecm} \times (\text{Mr2} / (\text{Mn} + \text{Mr2}));
\text{Er2cm} = \text{Ecm} \times (\text{Mn} / (\text{Mn} + \text{Mr2}));
\text{Vr2cm} = \sqrt{(2 \times \text{Mr2} \times \text{Er2cm}) / \text{Mr}};
\text{Vr2Lab} = \sqrt{(\text{Vcm}^2 + \text{Vr2cm}^2 - 2 \times \text{Vcm} \times \text{Vr2cm} \times \cos(3.142 - \text{th2Rad}))};
\text{Er2Lab} = 0.5 \times \text{Mr} \times (\text{Vr2Lab}^2);
\text{thRaDLab} = \arcsin((\text{Vr2cm} \times \sin(3.142 - \text{th2Rad})) / \text{Vr2Lab});
thDegLab = thRaDLab * 57.3

Vx2 = Vr2Lab * sin(thRaDLab);
Vz2 = Vr2Lab * cos(thRaDLab);

Vx2_rot = Vx2 * cos(thRad1tot) + Vz2 * sin(thRad1tot);
Vz2_rot = -Vx2 * sin(thRad1tot) + Vx2 * cos(thRad1tot);

thRad2tot = atan(Vx2_rot / Vz2_rot);

thDeg2tot = thRad2tot * 57.3

Vt2 = sqrt(Vx2^2 + Vz2^2);
Vt2_new = sqrt(Vx2_rot^2 + Vz2_rot^2);

Et2 = 0.5 * Mr2 * Vt2^2;

thxDeg2 = thDegLab;

E2(k) = Er2Lab;
Th2(k) = thxDeg2;

scatter(thDeg2tot, Er2Lab)

% figure
% plot(Th2, E2)

for m = 1:140,

th3Deg = (k - 140);

th3Rad = th2Deg / 57.3;

Ma1 = 4;
Ma2 = 4;
Q2 = 0.092;

Ecm = Et2 + Q2;
Ea1cm = Ecm * (Ma2 / (Ma1 + Ma2));
Ea2cm = Ecm * (Ma1 / (Ma1 + Ma2));

end
F Fortran Sort Code

c The Fortran Code Used For The Primary Data Analysis

*trigger
2048

*oned
1..128 adc1 4096
129 esum 4096
130 etot 4096
131 nf 80
132 nb 80
133 erel_12 4096
134 erel_13 4096
135 erel_23 4096
136 erel_5He_1 4096
137 erel_5He_2 4096
138 erel_5He_3 4096
139 erel_5He_4 4096
140 Windowed1 4096
141 Windowed2 4096
142 erelwindow1 4096
143 erelwindow2 4096
144 erelwindow12 4096
145 9Be_Ex_5He 4096
146 5He_1 4096
147 5He_2 4096
148 5He_3 4096
149 SingleSideEx 4096

*twod
1 adc 256
2 cadc 256
3 eth 256
4 efb1 256
5 efb2 256
6 efb3 256
subroutine init

implicit double precision (a−h,o−z)

save

include '.sunsort_initadc.i'

integer i,imap(16),n1f,n1b,n2f,n2b,n3f,n3b,n4f,n4b,np

integer imap1(16),imap2(16),in12,in13,in23,in12_2,in13_2,in23_2

integer*4 is

real ed(128),alpha(3,128),dum,m(128),o(128),cad(128),d1,d2

real x1(16),x2(16),y1(16),y2(16),e1f(16),e1b(16),e2f(16),e2b(16)

real thx1(16),thy1(16),thx2(16),thy2(16)

real er,prx,pry,prz

real x3(16),x4(16),y3(16),y4(16),e3f(16),e3b(16),e4f(16),e4b(16)

real thx3(16),thy3(16),thx4(16),thy4(16)

real e(128),px(128),py(128),pz(128),pxu,pny,pnz,pn,en

real th1,th2,th3,th4,esum,erel

real erel1,erel2,erel3,erel4,erel5,erel6,erel7

vars

s sort

c Declaration all variables used in the code
real th_tot(16)

integer np1,np2,np3,np4

real ymin,ymax,xmin,xmax

real r1

real r1x(16), r2x(16), r3x(16), r4x(16)

real alpha1

real phi

real QBe2

real Random(4)

real Ebeam,QBe,QHe

real thDeg, thRad

real Ma,Mr,Vb,Vbcm,Ebcm,Eacm,Etcem,Vacm,VaLab,EaLab

real thRaDLab,thDegLab,Etcem

real Ex,Ex1

real Emin,Emax

real e_3(3),px3(3),py3(3),pz3(3)

integer check,checks,check1,check2,check3,check4

logical in,inf

data imap/8,7,6,5,4,3,2,1,9,10,11,12,13,14,15,16/
data imap2/16,15,14,13,12,11,10,9,1,2,3,4,5,6,7,8/
data imap1/9,10,11,12,13,14,15,16,8,7,6,5,4,3,2,1/

is=0

c Detector positions and angles

d1=82
d2=109
d3=120
d4=80

a1=63.97/57.3

a2=27.72/57.3

a3=−30.00/57.3
a4 = -67.00/57.3

phi1 = 90.27/57.3
phi2 = 95.888/57.3
phi3 = 89.72/57.3
phi4 = 85.569/57.3

phi2,1 = 89.73/57.3
phi2,2 = 84.112/57.3
phi2,3 = 90.28/57.3
phi2,4 = 94.431/57.3

c Kinematics variables
Ebeam = 40.2
QBe = 0
QBe2 = -1.57

c Calibration process

c open calibration data file alpha.dat
open(unit=10, file='"/export/home/user/Documents/Year4Project/alpha2.dat")

do for all 64 channels
do i = 1, 128
read(10, *) dum, alpha(1,i), alpha(2,i), alpha(3,i)
c calc grad m using least squares fitting
E1 = 5.143
E2 = 5.49
E3 = 5.77
ECbar = (5.143*alpha(1,i) + 5.49*alpha(2,i) + 5.77*alpha(3,i))/3
Ebar = (5.143 + 5.49 + 5.77)/3
Cbar = (alpha(1,i) + alpha(2,i) + alpha(3,i))/3
C2bar = (alpha(1,i)*alpha(1,i) + alpha(2,i)*alpha(2,i) +
+ alpha(3,i)*alpha(3,i))/3
m(i) = (ECbar - Ebar*Cbar)/(C2bar - Cbar*Cbar)
c calc offset o
o(i) = Ebar - Cbar*m(i)
enddo
close(unit=10)
return
return

entry sortin

c Calculate calibrated values for all 128 channels

do i=1,128
cad(i)=0
if(adc(i).gt.10) then
cad(i)=real(adc(i))*m(i)+o(i)
call inc1d(i,nint(real(adc(i))))
call inc2d(1,i,nint(real(adc(i)/4.)))
call inc2d(2,i,nint(cad(i)+20.))
endif
enddo

c Undergo initial processing of the detected values in an event

c process detector 1

n1f=0
n1b=0
do i =1,16
if(cad(i).gt.1) then
n1f=n1f+1
e1f(n1f)=cad(i)
x1(n1f)=(real(imap(i))-8.5+(rand(is)-0.5))*50./16.
r1=sqrt(x1(n1f)**2+d1**2-x1(n1f)*d1*cos(90/57.3))
alpha1=asin(x1(n1f)*sin(90/57.3)/r1)
thy1(n1f)=alpha1
endif

if(cad(i+16).gt.1) then
n1b=n1b+1
e1b(n1b)=cad(i+16)
y1(n1b)=(real(imap(i))-8.5+(rand(is)-0.5))*50./16.
if (imap(i).gt.8) then
phi=phi1
else
phi=phi2
endif
\[ r_{1x}(n_{1b}) = \sqrt{y_{1}(n_{1b})^2 + d_{1}^2 - (2 + 2y_{1}(n_{1b})d_{1} \cos(\phi))^2} \]
\[ \alpha_{1} = \arcsin(y_{1}(n_{1b}) \sin(\phi) / r_{1x}(n_{1b})) \]
\[ \theta_{x1}(n_{1b}) = \alpha_{1} + \theta_{1} \]

\[ \text{endif} \]
\[ \text{enddo} \]

\text{c process detector 2}

\[ n_{2f} = 0 \]
\[ n_{2b} = 0 \]
\[ \text{do } i = 1, 16 \]
\[ \text{if } (\text{cad}(i+32) > 1) \text{ then} \]
\[ n_{2f} = n_{2f} + 1 \]
\[ e_{2f}(n_{2f}) = \text{cad}(i+32) \]
\[ x_{2}(n_{2f}) = \text{real} (\text{imap}(i)) - 8.5 + (\text{rand}(i) - 0.5) \times 50.0 / 16. \]
\[ r_{1} = \sqrt{x_{2}(n_{2f})^2 + d_{2}^2 - x_{2}(n_{2f})d_{2} \cos(90/57.3))} \]
\[ \alpha_{1} = \arcsin(x_{2}(n_{2f}) \sin(90/57.3) / r_{1}) \]
\[ \theta_{y_{2}}(n_{2f}) = \alpha_{1} \]
\[ \text{endif} \]
\[ \text{if } (\text{cad}(i+48) > 1) \text{ then} \]
\[ n_{2b} = n_{2b} + 1 \]
\[ e_{2b}(n_{2b}) = \text{cad}(i+48) \]
\[ y_{2}(n_{2b}) = \text{real} (\text{imap}(i)) - 8.5 + (\text{rand}(i) - 0.5) \times 50.0 / 16. \]
\[ \text{if } (\text{imap}(i) > 8) \text{ then} \]
\[ \phi = \phi_{2} \]
\[ \text{else} \]
\[ \phi = \phi_{2, 2} \]
\[ \text{endif} \]
\[ r_{2x}(n_{2b}) = \sqrt{y_{2}(n_{2b})^2 + d_{2}^2 - (2y_{2}(n_{2b})d_{2} \cos(\phi))^2} \]
\[ \alpha_{1} = \arcsin(y_{2}(n_{2b}) \sin(\phi) / r_{2x}(n_{2b})) \]
\[ \theta_{x2}(n_{2b}) = \alpha_{1} + \theta_{2} \]
\[ \text{endif} \]
\[ \text{enddo} \]

\text{c process detector 3}

\[ n_{3f} = 0 \]
\[ n_{3b} = 0 \]
do  i =1,16
   if(cad(i+64).gt.1) then
      n3f=n3f+1
      e3f(n3f)=cad(i+64)
      x3(n3f)=(real(imap(i))−8.5+(rand(is)−0.5))*50./16.
      r1=sqrt(x3(n3f)**2+d3**2−x3(n3f)*d3*cos(90/57.3))
      alpha1=asin(x3(n3f)*sin(90/57.3)/r1)
      thy3(n3f)=alpha1
   endif
   if(cad(i+80).gt.1) then
      n3b=n3b+1
      e3b(n3b)=cad(i+80)
      y3(n3b)=(real(imap(i))−8.5+(rand(is)−0.5))*50./16.
      if (imap(i).gt.8) then
         phi=phi3
      else
         phi=phi2_3
      endif
      r3x(n3b)=sqrt(y3(n3b)**2+d3**2−(2*y3(n3b)*d3*cos(phi)))
      alpha1=asin(y3(n3b)*sin(phi)/r3x(n3b))
      thx3(n3b)=alpha1+a3
   endif
endo
c process detector 4

n4f=0
n4b=0
do  i =1,16
   if(cad(i+96).gt.1.) then
      n4f=n4f+1
      e4f(n4f)=cad(i+96)
      x4(n4f)=(real(imap1(i))−8.5+(rand(is)−0.5))*50./16.
      r1=sqrt(x4(n4f)**2+d4**2−x4(n4f)*d4*cos(90/57.3))
      alpha1=asin(x4(n4f)*sin(90/57.3)/r1)
      thy4(n4f)=alpha1
   endif
if (cad(i+112).gt.1.) then
  n4b=n4b+1
  e4b(n4b)=cad(i+112)
  y4(n4b)=(real(imap1(i))−8.5+(rand(is)−0.5))*50./16.
  if (imap1(i).lt.9) then
    phi=phi4
  else
    phi=phi2,4
  endif
  r4x(n4b)=sqrt(y4(n4b)**2+d4**2−(2*y4(n4b)*d4*cos(phi)))
  alpha1=−asin(y4(n4b)*sin(phi)/r4x(n4b))
  thx4(n4b)=alpha1+a4
endif
enddo

c Plot amount on front and back of each detector as a check

  call inc1d(131,n1f)
  call inc1d(131,n2f+16)
  call inc1d(131,n3f+32)
  call inc1d(131,n4f+48)
  call inc1d(132,n1b)
  call inc1d(132,n2b+16)
  call inc1d(132,n3b+32)
  call inc1d(132,n4b+48)

c energy order the front and back strips

  call order(n1f,e1f,x1)
  call order(n2f,e2f,x2)
  call order(n3f,e3f,x3)
  call order(n4f,e4f,x4)
  call order2(n1b,e1b,thx1,r1x)
  call order2(n2b,e2b,thx2,r2x)
  call order2(n3b,e3b,thx3,r3x)
  call order2(n4b,e4b,thx4,r4x)
c Plotting the theorised scattered beam energy vs scattering angle

Ma=4.00151  
Mr=9.0122  
thDeg=-110

c Uncomment the do loop and plotting functions to plot the lines

do thDeg=0,100

        Mr=9.0122

        thRad=thDeg/57.3

        Vb=sqrt((2*Ma*Ebeam)/Ma)

        Vcm=Vb*(Ma/(Ma+Mr))

        Vbcm=sqrt((2*Ma*Ebeam)*(Mr/(Ma*(Ma+Mr))))

        Ebcm=0.5*Ma*(Vbcm**2)

        Etcm=0.5*Mr*(Vcm**2)

        Ecm=Ebcm+Etcm

        Eacm=Ecm*(Mr/(Ma+Mr))

        Ercm=Ecm*(Ma/(Ma+Mr))

        Vacm=sqrt((2*Ma*Eacm)/Ma)

        VaLab=sqrt((Vcm**2 + Vacm**2 - 2*Vcm*Vacin*cos(3.142*abs(thRad))))

        EaLab=0.5*Ma*(VaLab**2)

        thRaDLab=asin(Vacm*sin(thRad)/VaLab)

        thDegLab=thRaDLab*57.3

c call inc2d(3,nint(thDegLab+128),nint(EaLab+5))

Mr=12.0

thRad=thDeg/57.3

Vb=sqrt((2*Ma*Ebeam)/Ma)

Vcm=Vb*(Ma/(Ma+Mr))

Vbcm=sqrt((2*Ma*Ebeam)*(Mr/(Ma*(Ma+Mr))))

Ebcm=0.5*Ma*(Vbcm**2)

Etcm=0.5*Mr*(Vcm**2)

Ecm=Ebcm+Etcm

Eacm=Ecm*(Mr/(Ma+Mr))
Ercm = Ecm * (Ma / (Ma + Mr))

Vacm = sqrt(2 * Ma * Eacm) / Ma

VaLab = sqrt(Vcm**2 + Vacm**2 - 2 * Vcm * Vacm * cos(3.142 - abs(thRad)))

EaLab = 0.5 * Ma * (VaLab**2)

thRaDLab = asin(Vacm * sin(thRad) / VaLab)

thDegLab = thRaDLab * 57.3

c call inc2d(3, nint(thDegLab + 128), nint(EaLab + 5))

Mr = 16.0

thRad = thDeg / 57.3

Vb = sqrt(2 * Ma * Ebeam) / Ma

Vcm = Vb * (Ma / (Ma + Mr))

Vbcm = sqrt(2 * Ma * Ebeam * (Mr / (Ma * (Ma + Mr))))

Ebcm = 0.5 * Ma * (Vbcm**2)

Etcm = 0.5 * Mr * (Vcm**2)

Ecm = Ebcm + Etcm

Eacm = Ecm * (Mr / (Ma + Mr))

Ercm = Ecm * (Ma / (Ma + Mr))

VaLab = sqrt(Vcm**2 + Vacm**2 - 2 * Vcm * Vacm * cos(3.142 - abs(thRad)))

EaLab = 0.5 * Ma * (VaLab**2)

thRaDLab = asin(Vacm * sin(thRad) / VaLab)

thDegLab = thRaDLab * 57.3

c call inc2d(3, nint(thDegLab + 128), nint(EaLab + 5))

c enddo

c-----------------------------------------------------------------------------------------------------------------

c Plots the energy vs angle 2D plots

th1(1) = atan(x1(1)/r1x(1))

th1 = acos(cos(thx1(1)) * cos(thy1(1))) * 57.3 * thx1(1) / abs(thx1(1))

if (n1f.eq.1.and.n1b.eq.1) + call inc2d(3, nint(th1 + 128), nint(e1f(1) + 5))

th2(1) = atan(x2(1)/r2x(1))

th2 = acos(cos(thx2(1)) * cos(thy2(1))) * 57.3 * thx2(1) / abs(thx2(1))

if (n2f.eq.1.and.n2b.eq.1)
+ call inc2d(3,nint(th2+128),nint(e2f(1)*5))
thy3(1) = atan(x3(1)/r3x(1))
th3=acos(cos(thx3(1))*cos(thy3(1)))+57.3*thx3(1)/abs(thx3(1))
if(n3f.eq.1) call inc2d(3,nint(th3+128),nint(e3f(1)*5))
thy4(1) = atan(x4(1)/r4x(1))
th4=acos(cos(thx4(1))*cos(thy4(1)))+57.3*thx4(1)/abs(thx4(1))
if(n4f.ge.1.and.n4f.ge.1) + call inc2d(3,nint(th4+128),nint(e4f(1)*5))
if(n1f.eq.1) call inc2d(4,nint(e1f(1)*10.),nint(e1b(1)*10.))
if(n2f.eq.1) call inc2d(5,nint(e2f(1)*10.),nint(e2b(1)*10.))
if(n3f.eq.1) call inc2d(6,nint(e3f(1)*10.),nint(e3b(1)*10.))
if(n4f.ge.1) call inc2d(7,nint(e4f(1)*10.),nint(e4b(1)*10.))
if(n1f.eq.1) + call inc2d(10,nint(thx1(1)+57.3+128),nint(thy1(1)+57.3+128))
if(n2f.eq.1) + call inc2d(10,nint(thx2(1)+57.3+128),nint(thy2(1)+57.3+128))
if(n3f.eq.1) + call inc2d(10,nint(thx3(1)+57.3+128),nint(thy3(1)+57.3+128))
if(n4f.eq.1) + call inc2d(10,nint(thx4(1)+57.3+128),nint(thy4(1)+57.3+128))

Further process each detector. Calculate the angle of detection
Group all detections into a single array for further analysis
np=0
np1=0
np2=0
np3=0
np4=0
if(n1f.ge.1) then
  do i=1,min(n1f,n1b)
    if(abs(e1f(i)-e1b(i)).lt.0.1) then
      np= np+1
    np1= np1+1
thy1(i) = atan(x1(i)/r1x(i))
e(np)=e1f(i)
px(np)=sqrt(2*e(np)*sin(thx1(i))*cos(th1(i)))
py(np)=sqrt(2*e(np)*cos(th1(i)))*cos(thy1(i))

th_tot(np)=acos(cos(thx1(i)))*cos(th1(i)))*57.3*thx1(i)/abs(thx1(i))
endif
enddo
endif

if(n2f.ge.1) then
    do i=1,min(n2f,n2b)
        if(abs(e2f(i)-e2b(i)).lt.0.1) then
            np=np+1
            np2=np2+1
            thy2(i) = atan(x2(i)/r2x(i))
e(np)=e2f(i)
px(np)=sqrt(2*e(np)*sin(thx2(i))*cos(thy2(i)))
py(np)=sqrt(2*e(np)*cos(thy2(i)))*cos(th2(i))

th_tot(np)=acos(cos(thx2(i)))*cos(thy2(i)))*57.3*thx2(i)/abs(thx2(i))
endif
enddo
endif
endif

if(n3f.ge.1) then
    do i=1,min(n3f,n3b)
        if(abs(e3f(i)-e3b(i)).lt.0.1) then
            np=np+1
            np3=np3+1
            thy3(i) = atan(x3(i)/r3x(i))
e(np)=e3f(i)
px(np)=sqrt(2*e(np)*sin(thx3(i))*cos(thy3(i)))
py(np)=sqrt(2*e(np)*sin(thy3(i)))*cos(th3(i))
px(np)=sqrt(2*e(np)*cos(th3(i)))*cos(thy3(i))

th_tot(np)=acos(cos(thx3(i)))*cos(thy3(i)))*57.3*thx3(i)/abs(thx3(i))
endif
enddo
endif
endif

if(n4f.ge.1) then
do i=1,min(n4f,n4b)
  if(abs(e4f(i)-e4b(i)).lt.0.1) then
    np=np+1
    np4=np4+1
    thy4(i) = atan(x4(i)/r4x(i))
    e(np)=e4f(i)
    px(np)=sqrt(2*4.0*e(np))*sin(thx4(i))*cos(thy4(i))
    py(np)=sqrt(2*4.0*e(np))*sin(thy4(i))
    pz(np)=sqrt(2*4.0*e(np))*cos(thx4(i))*cos(thy4(i))
    th_tot(np)=acos(cos(thx4(i))*cos(thy4(i)))*57.3*thx4(i)/abs(thx4(i))
  endif
enddo
endif
!
!
c Order all arrays of detections in angle order
!
call order2(np,th_tot,e,px)
call order2(np,th_tot,py,pz)
!
c This section plots the sum energy spectrum for the decay to check for

c events that come from 9Be for all 3 particle events

c Also calculates the Q−value for the decay of intermediate 8Be and draws

c the Dalitz plots
!
if (np.eq.3) then
  esum=e(1)+e(2)+e(3)
call inc1d(129,nint(esum*10.))
  pnx=-(px(1)+px(2)+px(3))
  pny=-(py(1)+py(2)+py(3))
  pnz=sqrt(2*4.0*Ebeam)-(px(1)+pz(2)+pz(3))
  pn=sqrt(pnx**2+pny**2+pnz**2)
  en=pn**2/(2.*1.)
call inc1d(130,nint((esum+en−QBe2)*10.))
in12=0
in13=0
in23=0
in12_2=0
in13_2=0
in23,2=0
if((esum+en−QBe2)*10.lt.462.3.and.(esum+en−QBe2)*10.gt.345.3) then
   call er8Be(e,px,py,pz,1,2,erel1)
call inc1d(133,nint(erie1*100.))
   if(erie1.lt.0.12) in12=1
   if(erie1.lt.1) in12=1
   call er8Be(e,px,py,pz,1,3,erie2)
call inc1d(134,nint(erie2*100.))
   if(erie2.lt.0.12) in13=1
   if(erie2.lt.4) in13=1
   call er8Be(e,px,py,pz,3,2,erie3)
call inc1d(135,nint(erie3*100.))
   if(erie3.lt.0.12) in23=1
   if(erie3.lt.4) in23=1
   call inc2d(12,nint(erie1*10),nint(erie3*10))
call inc2d(13,nint(erie1*10),nint(erie2*10))
call inc2d(14,nint(erie2*10),nint(erie3*10))
c
This bit triggers on certain areas of the 2D Dalitz plot

c Dalitz plot windowing boundaries are defined below.
c Uncomment the ones you want to use

c GS Boundaries
ymin=6.75
ymax=25
xmin=0.0806
xmax=0.108

c Lower 2+ boundaries
c ymin=6.75
c ymax=25
c xmin=0.38735
c xmax=0.735

c 2+ boundaries
c ymin=6.75
c ymax=25
c xmin=1.307
check=0

check=0

c Checks if in window

call window(erel1,erel3,xmin,xmax,ymin,ymax,check)

c If particles 1 and 2 come from a state in 8Be, the do the following

if (check.eq.1) then

call inc1d(140,nint(erel1*300+100))

prx=-(px(3))

pry=-(py(3))

prz=sqrt((2*4*Ebeam)-(pz(3)))

pr=sqrt(prx**2+pry**2+prz**2)

er=pr**2/(2.*9.)

call inc2d(17,nint(th*57.3+128),nint((Ex1)*8))

call inc2d(18,nint(erel1*40),nint((Ex1)*10))

call inc1d(142,nint((e(3)+er)*50)+100))

call inc1d(144,nint(((Ex1)*10.)))

endif

c If particles 2 and 3 come from a state in 8Be, the do the following

Ex1=Ebeam+QBe-e(3)-er

th=acos(px(3)/sqrt(px(3)**2+py(3)**2+pz(3)**2))*px(3)/abs(px(3))

call inc2d(17,nint(th*57.3+128),nint((Ex1)*8))

call inc2d(18,nint(erel1*40),nint((Ex1)*10))

call inc1d(142,nint((e(3)+er)*50)+100))

call inc1d(144,nint(((Ex1)*10.)))
if (check.eq.2) then
  call inc1d(141,nint(erelel3*300+100))
  prx=-(px(1))
  pry=-(py(1))
  prz=sqrt(2*4*Ebeam)-(pz(1))
  pr=sqrt(prx**2+pry**2+prz**2)
  er=pr**2/(2.*9.)
  Ex1=Ebeam+QBe-e(1)-er
  Calculate excitation energy
  th=acos(pz(1)/sqrt(px(1)**2+py(1)**2+pz(1)**2))*px(1)/abs(px(1))
  call inc2d(17,nint((th*57.3+128),nint((Ex1)*8)))
  call inc2d(18,nint(erelel3*40),nint((Ex1)*10))
  call inc1d(143,nint(((e(1)+er)+50)+100))
  call inc1d(144,nint(((Ex1)*10.)))
  endif

!---------------------------------------------------------------

! *** Now unused section ***

! calc Etot from each of the three particles

! When looking at supposed recoil alpha 1, then check if
! particles 2 and 3 were detected within the 8Be gs peak.
! If so, then plot the Q−value data in 2D plot 11.

! If the event lies outside (higher) than the 8Be gs then
! plot the data in 2D plot 9

! If it lies below the 8Be gs then plot in 2D plot 8

  prx=-(px(1))
  pry=-(py(1))
  prz=sqrt(2*4*Ebeam)-(pz(1))
  pr=sqrt(prx**2+pry**2+prz**2)
  er=pr**2/(2.*9.)
  th=acos(pz(1)/sqrt(px(1)**2+py(1)**2+pz(1)**2))*px(1)/abs(px(1))
  if(in23.eq.1) call inc2d(8,nint(th*57.3+128),nint((e(1)+er)*5))
  if(in23.eq.0) call inc2d(9,nint(th*57.3+128),nint((e(1)+er)*5))
  if(inc2eq.0 .and. in23.eq.1)
    call inc2d(11,nint(th*57.3+128),nint((e(1)+er)*5))
prx=-(px(2))
pry=-(py(2))
prz=sqrt(2*4+Ebeam)-(pz(2))
pr=sqrt(prx**2+pry**2+prz**2)
er=pr**2/(2.*9.)

th=acos(pz(2)/sqrt(px(2)**2+py(2)**2+pz(2)**2))*px(2)/abs(px(2))

if[in13.eq.1] call inc2d(8,nint(th*57.3+128),nint((e(2)+er)*5))
if[in13.eq.0] call inc2d(9,nint(th*57.3+128),nint((e(2)+er)*5))
if[in13.eq.0 .and. in13.eq.1]
  + call inc2d(11,nint(th*57.3+128),nint((e(2)+er)*5))
endif
endif

prx=-(px(3))
pry=-(py(3))
prz=sqrt(2*4+Ebeam)-(pz(3))
pr=sqrt(prx**2+pry**2+prz**2)
er=pr**2/(2.*9.)

th=acos(pz(3)/sqrt(px(3)**2+py(3)**2+pz(3)**2))*px(3)/abs(px(3))

if[in12.eq.1] call inc2d(8,nint(th*57.3+128),nint((e(3)+er)*5))
if[in12.eq.0] call inc2d(9,nint(th*57.3+128),nint((e(3)+er)*5))
if[in12.eq.0 .and. in12.eq.1]
  + call inc2d(11,nint(th*57.3+128),nint((e(3)+er)*5))
endif
endif

c This calculates excitation of proposed 5He

ymin=34
ymax=50
xmin=8
xmax=250
Emin=4.6
EMax=6.9

if (np.eq.3) then
  esum=e(1)+e(2)+e(3)
pnx=-(px(1)+px(2)+px(3))
pny=-(py(1)+py(2)+py(3))
endif
\[ \text{pnz} = \sqrt{2 \cdot 4 \cdot E_{\text{beam}}} - (p_x(1) + p_x(2) + p_x(3)) \]

\[ \text{pn} = \sqrt{p_n^2 + p_y^2 + p_z^2} \]

\[ \text{en} = p_n^2 / (2 \cdot 1) \]

\[ \text{If event comes from } 9\text{Be then calculate } 5\text{He decay Q values when pairing each alpha with the neutron. Plot the Dalitz Plots} \]

\[ \text{if} \ (\text{esum} + \text{en} - Q_{\text{Be}2} < 10.1 \cdot 462.3 \text{and} (\text{esum} + \text{en} - Q_{\text{Be}2} > 10.1 \cdot 345.3) \text{ then} \]

\[ \text{call er5He(e,px,py,pz,pnx,pny,pnz,1,en,erel5)} \]

\[ \text{call er5He(e,px,py,pz,pnx,pny,pnz,2,en,erel6)} \]

\[ \text{call er5He(e,px,py,pz,pnx,pny,pnz,3,en,erel7)} \]

\[ \text{call inc1d(146,nint(erel5*100.))} \]

\[ \text{call inc1d(147,nint(erel6*100.))} \]

\[ \text{call inc1d(148,nint(erel7*100.))} \]

\[ \text{call inc2d(24,nint((erel5*8.)+10),nint((erel6*8.)+10))} \]

\[ \text{call inc2d(25,nint((erel6*8.)+10),nint((erel7*8.)+10))} \]

\[ \text{call inc2d(26,nint((erel5*8.)+10),nint((erel7*8.)+10))} \]

\[ \text{check}=0 \]

\[ \text{check1}=0 \]

\[ \text{check2}=0 \]

\[ \text{check3}=0 \]

\[ \text{check4}=0 \]

\[ \text{Parts that plot } 8\text{Be Q value vs } 5\text{He Q value Dalitz plots for different pairings} \]

\[ \text{if } (\text{np1}+\text{np2} eq 2 \text{and} \text{np3}+\text{np4} eq 1) \text{ then} \]

\[ \text{call er8Be(e,px,py,pz,1,2,erel4)} \]

\[ \text{call er5He(e,px,py,pz,pnx,pny,pnz,1,en,erel1)} \]

\[ \text{call inc1d(136,nint(erel1*100.))} \]

\[ \text{call inc2d(15,nint(erel1*5),nint(erel4*20))} \]

\[ \text{call er5He(e,px,py,pz,pnx,pny,pnz,2,en,erel2)} \]

\[ \text{call inc1d(137,nint(erel2*100.))} \]

\[ \text{call inc2d(16,nint(erel2*5),nint(erel4*20))} \]

\[ \text{endif} \]
if (np1+np2.eq.1.and.np3+np4.eq.2) then
    call er8Be(e,px,py,pz,2,3,erel4)
    call er5He(e,px,py,pz,pxx,pyy,prz,2,en,erel2)
    call inc1d(138,nint((erel2*100.)))
    call inc2d(19,nint(erel2*5),nint(erel4*20))
    call er5He(e,px,py,pz,pxx,pyy,prz,3,en,erel3)
    call inc1d(139,nint((erel3*100.)))
    call inc2d(20,nint(erel3*5),nint(erel4*20))
endif
endif
endif
return
entry finish
return
end

--
c subroutine order arranges the energy values in the array e in an increasing order
subroutine order (n,e,p)
integer n
real e(16),p(16),tmp(16)
do i=1,n
do j=1,i-1
tmp(1) = e(i)
tmp(2) = e(j)

tmp(3) = p(i)

tmp(4) = p(j)

if(e(i).gt.e(j)) then

e(i)=tmp(2)

e(j)=tmp(1)

p(i)=tmp(4)

p(j)=tmp(3)
endif
endo
do
endo
c return
dend

c subroutine order aranges the energy values in the array e in an increasing order

subroutine order2(n,e,p,q)

integer n

real e(16),p(16),tmp(16), q(16)

do i=1,n
do j=1,i-1

tmp(1) = e(i)

tmp(2) = e(j)

tmp(3) = p(i)

tmp(4) = p(j)

tmp(5) = q(i)


tmp(6) = q(j)
if(e(i).gt.e(j)) then

e(i)=tmp(2)

e(j)=tmp(1)

p(i)=tmp(4)

p(j)=tmp(3)

q(i)=tmp(6)
endif
endo
q(j)=tmp(5)
endif
enddo
dodo
return
end
c Calculates Q value of 8Be decay
subroutine er8Be(e,px,py,pz,p1,p2,erel)
real px(128),py(128),pz(128),e(128),pbx,pby,pbz,pbe,ebe,erel
integer p1,p2

pbx=px(p1)+px(p2)
pby=py(p1)+py(p2)
pbx=pz(p1)+pz(p2)
pbe=sqrt(pbx**2+pby**2+pbz**2)
ebe=pbe**2/(2.*8.)
erel=e(p1)+e(p2)-ebe
return
dend

c Calculates Q value of 5He decay
subroutine er5He(e,px,py,pz,pnx,pny,pnz,p1,en,erel)
real px(128),py(128),pz(128),e(128),pnx,pny,pnz,phx,phy,phz,phe
real ehe,erel,en
integer p1

phx=px(p1)+pnx
phy=py(p1)+pny
phz=pz(p1)+pnz
pbe=sqrt(phx**2+phy**2+phz**2)
ehe=phe**2/(2.*5.)
erel=e(p1)+en-ehe
return
end

c Checks if the event is within a $^{8}\text{Be}$ Dalitz plot window

```fortran
subroutine window(erel1, erel3, xmin, xmax, ymin, ymax, check)
real erel1, erel3, xmin, xmax, ymin, ymax
integer check

check=0

if (erel1.gt.xmin.and.erel1.lt.xmax) then
  if (erel3.gt.ymin.and.erel3.lt.ymax) then
    check=1
  endif
endif

if (erel3.gt.xmin.and.erel3.lt.xmax) then
  if (erel1.gt.ymin.and.erel1.lt.ymax) then
    check=2
  endif
endif

return
end
```

c Unused. Checks if the event is within a $^{5}\text{He}$ Dalitz plot window

```fortran
subroutine window5He(erel1, erel3, xmin, xmax, ymin, ymax, check)
real erel1, erel3, xmin, xmax, ymin, ymax
integer check

check=0

if (erel1*5+10.gt.xmin.and.erel1*5+10.lt.xmax) then
  if (erel3*20+10.gt.ymin.and.erel3*20+10.lt.ymax) then
    check=1
  endif
endif

end
```
G  Peak Fitting

The following section contains the main code and functions used for peak fitting to the excitation spectra.

G.1  Single Voigt Plotter

```
function output= VoigtSingle(p,input)
    % this is the 3 lorentzian equations here:
    output = p(3)*pi+p(2)/2+real(faddeeva(((input−p(1)) + 1i*p(2)/2)/(p(4)*sqrt(2))),15))/(p(4)*sqrt(2*pi));
```

G.2  Multiple Voigt Function

```
function output= myPolyCurveVoigtFinalGeneral(p,input)

    Size=size(p);
    Max=Size(2);
    maxiter=((Size(2)−1)/3)−1;
    Step=3;
    Answer=p(3)*pi+p(2)/2+real(faddeeva(((input−p(1)) +... 1i*p(2)/2)/(p(Max)*sqrt(2))),15))/(p(Max)*sqrt(2*pi));

    for k=1:maxiter,
        Answer=Answer+( p(3+(Step*k))*pi+p(2+(Step*k))/2*real(faddeeva(((input−p(1+(Step*k)))) +... 1i*p(2+(Step*k))/2)/(p(Max)*sqrt(2))),15))/(p(Max)*sqrt(2*pi)) ;
    end

    output=Answer;
```

G.3  Fadeeva Function

The Fadeeva function used in the Voigt profile definition [19]

```
function w = faddeeva(z,N)
```
% FADDEEVA Faddeeva function
% W = FADDEEVA(Z) is the Faddeeva function, aka the plasma dispersion
% function, for each element of Z. The Faddeeva function is defined as:
% %
% 7 w(z) = exp(−z^2) * erfc(jz)
% % where erfc(x) is the complex complementary error function.
% % W = FADDEEVA(Z,N) can be used to explicitly specify the number of terms
% to truncate the expansion (see (13) in [1]). N = 16 is used as default.
% %
% Example:
% x = linspace(-10,10,1001); [X,Y] = meshgrid(x,x);
% W = faddeeva(complex(X,Y));
% figure;
% subplot(121); imagesc(x,x,real(W)); axis xy square; caxis([-1 1]);
% title('re(faddeeva(z))'); xlabel('re(z)'); ylabel('im(z)');
% subplot(122); imagesc(x,x,imag(W)); axis xy square; caxis([-1 1]);
% title('im(faddeeva(z))'); xlabel('re(z)'); ylabel('im(z)');

if nargin<2, N = []; end
if isempty(N), N = 16; end

w = zeros(size(z)); % initialize output

% for purely imaginary—valued inputs, use erf as is if z is real
idx = real(z)==0; %
w(idx) = exp(-z(idx).^2).erfc(imag(z(idx)));

if all(idx), return; end
idx = ~idx;

% for complex—valued inputs

% make sure all points are in the upper half—plane (positive imag. values)
idx1 = idx & imag(z)<0;
z(idx1) = conj(z(idx1));

M = 2*N;
G.4 Least Squares Fitting Matlab Code

M2 = 2*M;

k = (-M+1:1:M-1); \% M2 = no. of sampling points.

L = sqrt(N/sqrt(2)); \% Optimal choice of L.

\[ \theta = k \times \frac{\pi}{M}; \]

\[ t = L \times \tan \left( \frac{\theta}{2} \right); \% Variables \theta \text{ and } t. \]

\[ f = \exp(-t^2) \times (L^2 + t^2); \]

\[ f = [0; f]; \% Function to be transformed. \]

\[ a = \text{real}(\text{fft}(\text{fltsift}(f)))/M2; \% Coefficients of transform. \]

\[ a = \text{flipud}(a(2:N+1)); \% Reorder coefficients. \]

\[ Z = (L+1i \times z(idx))/(L-1i \times z(idx)); \]

\[ p = \text{polyval}(a,Z); \% Polynomial evaluation. \]

\[ w(idx) = 2p/((L-1i \times z(idx))^2 + (1/sqrt(\pi))/(L-1i \times z(idx))); \% Evaluate \ w(z). \]

\% convert the upper half–plane results to the lower half–plane if necessary

\[ w(idx1) = \text{conj}(2 \times \exp(z(idx1)^2) \times w(idx1)); \]

G.4 Least Squares Fitting Matlab Code

\[ \text{clear all;} \]
\[ \text{close all;} \]

\% Read in 9Be data

loreztest = csvread('9BeGS_smallbin.csv');

\% Read in carbon spectrum

carbon = csvread('12CGS_scale.csv');

Scale = 0.08287;

Yscale = (carbon(:,2)) * Scale;

\%Yscale=0

\% Read in the bounds for the fit parameters

bounds = csvread('BoundsRestrictIsospinAll4_NewStates.csv')
G.4 Least Squares Fitting Matlab Code

22 lb = bounds(:,1)';
23 ub = bounds(:,2)';

% 1st column vector for loreztest
25 X = loreztest(:,1)';
26
27 %%%% plot all the raw data on the same figure %%%% figure
28 Graph = plot(X,Y1)
29 set(Graph,'Color','blue','LineWidth',1)
30 xlabel('Ex (MeV)','FontSize',14)
31 ylabel('Number of Events','FontSize',14)
32 title('9Be Excitation Spectrum: 8Be Low 2+ Decay Mode','FontSize',14)
33
34 Y = (loreztest(:,2)')
35
36 %%%%%%%%%%%%%%%%%%%%%
37 %%%%% Plot the carbon Spectrum Scaled and unscaled
38 figure
39 Graph1 = plot(X,Yscale)
40 xlabel('Ex (MeV)','FontSize',14)
41 ylabel('Number of Events','FontSize',14)
42 title('Scaled 12C Excitation Spectrum: 8Be 2+ Decay Mode','FontSize',14)
43
44 figure
45 Graph2 = plot(X,(Yscale)/Scale)
46 xlabel('Ex (MeV)','FontSize',14)
47 ylabel('Number of Events','FontSize',14)
48 title('Unscaled 12C Excitation Spectrum: 8Be 2+ Decay Mode','FontSize',14)
49
50 %%%%%%%%%%%%%%%%%%%%%
51 %%%% plot all the corrected data on the same figure %%%% figure
52
G.4 Least Squares Fitting Matlab Code

```matlab
Graph = plot(X,Y)
%Graph = plot(X,Y,4,'g','filled')
set(Graph,'Color','green','LineWidth',1)
xlabel('Ex (MeV)','FontSize',14)
ylabel('Number of Events','FontSize',14)
title('9Be Excitation Spectrum: 8Be GS Decay Mode','FontSize',14)
hold
%hold the figure
initialConditions = (ub + lb)/2
%configure the optimset for use with lsqcurvet
options = optimset('lsqcurvet');
%increase the number of function evaluations for more accuracy
options.MaxFunEvals = 5000;
[newParameters2,error2] = lsqcurvet(@myPolyCurveMultiVoigtFinal, initialConditions,X,Y,lb,ub,options);
%use new parameters to get new output values
Y3 = myPolyCurveMultiVoigtFinalGeneral(newParameters2,X);
%plot the new data using the color blue
plot(X,Y3,'b')
legend('Data','Least Squares Fit')
Step=size(newParameters2)
Step2=Step(2)
Max=Step2/3
for j=1:Max,
%peak=num2str(Step);
peak=num2str(j);
['peak ' peak]
disp('Parameters')
parameters=newParameters2([1+(3*(j-1)) 2+(3*(j-1)) 3+(3*(j-1)) Step2])
y=VoigtSingle(parameters,X);
Area = trapz(X,y)
plot(X,y,'r')
end
```

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G.5 Parameter Boundaries Input File

The initial guesses for the fit parameters of each peak are inputted into the code through a tabulated .csv file in the format shown in figure 58 below. The true input file contains no table headings.
Figure 58: Table showing the input format of the fit parameters’ upper and lower bounds.

H Penetrability Calculations

H.1 Barrier Penetrability Calculations: New State 1

The barrier penetrability calculations for the first new state are shown in figure 59 below.

Figure 59: Table showing the theorised barrier penetrabilities and relative branching ratios for decays of the first new state in $^9$Be into the $^8$Be$^{9+}$ and $^8$Be$^{2+}$ states
H.2 Barrier Penetrability Calculations: New State 2

The barrier penetrability calculations for the second new state are shown in figure 60 below.

<table>
<thead>
<tr>
<th>States in $^{9}$Be</th>
<th>$L \rightarrow 0^+$ Penetrability</th>
<th>$^8$Be $L \rightarrow 2^+$ Penetrability</th>
<th>Ratio $0^+/2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{1/2}^+$</td>
<td>0</td>
<td>2.6980</td>
<td>1.4329</td>
</tr>
<tr>
<td>$^{3/2}^+$</td>
<td>2</td>
<td>1.9520</td>
<td>2.4766</td>
</tr>
<tr>
<td>$^{5/2}^+$</td>
<td>2</td>
<td>1.9520</td>
<td>2.4766</td>
</tr>
<tr>
<td>$^{7/2}^+$</td>
<td>4</td>
<td>0.3230</td>
<td>1.4329</td>
</tr>
<tr>
<td>$^{9/2}^+$</td>
<td>4</td>
<td>0.3230</td>
<td>1.4329</td>
</tr>
<tr>
<td>$^{11/2}^+$</td>
<td>6</td>
<td>0.0042</td>
<td>0.0211</td>
</tr>
<tr>
<td>$^{13/2}^+$</td>
<td>6</td>
<td>0.0042</td>
<td>0.0211</td>
</tr>
<tr>
<td>$^{15/2}^+$</td>
<td>1</td>
<td>2.5900</td>
<td>2.1294</td>
</tr>
<tr>
<td>$^{17/2}^+$</td>
<td>1</td>
<td>2.5900</td>
<td>2.1294</td>
</tr>
<tr>
<td>$^{19/2}^+$</td>
<td>3</td>
<td>1.0610</td>
<td>2.1294</td>
</tr>
<tr>
<td>$^{21/2}^+$</td>
<td>3</td>
<td>1.0610</td>
<td>2.1294</td>
</tr>
<tr>
<td>$^{23/2}^+$</td>
<td>5</td>
<td>0.0490</td>
<td>0.5968</td>
</tr>
<tr>
<td>$^{25/2}^+$</td>
<td>5</td>
<td>0.0490</td>
<td>0.5968</td>
</tr>
<tr>
<td>$^{27/2}^+$</td>
<td>7</td>
<td>0.0002</td>
<td>0.0117</td>
</tr>
</tbody>
</table>

Figure 60: Table showing the theorised barrier penetrabilities and relative branching ratios for decays of the second new state in $^{9}$Be into the $^8$Be$^0$ and $^8$Be$^{2+}$ states

H.3 Barrier Penetrability Calculations: Existing $\approx 11.2$ MeV State

The barrier penetrability calculations for the known state at $\approx 11.2$ MeV are shown in figure 61 below.

<table>
<thead>
<tr>
<th>States in $^{9}$Be</th>
<th>$L \rightarrow 0^+$ Penetrability</th>
<th>$^8$Be $L \rightarrow 2^+$ Penetrability</th>
<th>Ratio $0^+/2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{1/2}^+$</td>
<td>0</td>
<td>2.6995</td>
<td>1.1366</td>
</tr>
<tr>
<td>$^{3/2}^+$</td>
<td>2</td>
<td>1.7073</td>
<td>2.2342</td>
</tr>
<tr>
<td>$^{5/2}^+$</td>
<td>2</td>
<td>1.7073</td>
<td>2.2342</td>
</tr>
<tr>
<td>$^{7/2}^+$</td>
<td>4</td>
<td>0.2094</td>
<td>1.1366</td>
</tr>
<tr>
<td>$^{9/2}^+$</td>
<td>4</td>
<td>0.2094</td>
<td>1.1366</td>
</tr>
<tr>
<td>$^{11/2}^+$</td>
<td>6</td>
<td>0.0019</td>
<td>0.0575</td>
</tr>
<tr>
<td>$^{13/2}^+$</td>
<td>6</td>
<td>0.0019</td>
<td>0.0575</td>
</tr>
<tr>
<td>$^{15/2}^+$</td>
<td>1</td>
<td>2.3758</td>
<td>1.8613</td>
</tr>
<tr>
<td>$^{17/2}^+$</td>
<td>1</td>
<td>2.3758</td>
<td>1.8613</td>
</tr>
<tr>
<td>$^{19/2}^+$</td>
<td>3</td>
<td>0.8301</td>
<td>1.8613</td>
</tr>
<tr>
<td>$^{21/2}^+$</td>
<td>3</td>
<td>0.8301</td>
<td>1.8613</td>
</tr>
<tr>
<td>$^{23/2}^+$</td>
<td>5</td>
<td>0.0260</td>
<td>0.3841</td>
</tr>
<tr>
<td>$^{25/2}^+$</td>
<td>5</td>
<td>0.0260</td>
<td>0.3841</td>
</tr>
<tr>
<td>$^{27/2}^+$</td>
<td>7</td>
<td>0.0001</td>
<td>0.0043</td>
</tr>
</tbody>
</table>

Figure 61: Table showing the theorised barrier penetrabilities and relative branching ratios for decays of the existing state at 11.2 MeV in $^{9}$Be into the $^8$Be$^0$ and $^8$Be$^{2+}$ states

H.4 Barrier Penetrability Calculations: Existing $\approx 11.8$ MeV State

The barrier penetrability calculations for the known state at $\approx 11.8$ MeV are shown in figure 62 below.
Table showing the theorised barrier penetrabilities and relative branching ratios for decays of the existing state at 11.8 MeV in $^9$Be into the $^8$Be$^{gs}$ and $^8$Be$^{2+}$ states

10  References

References


REFERENCES


