DESIGN, ANALYSIS, AND TEST OF SYNTHETIC JET ACTUATOR FOR SMALL UNMANNED AERIAL VEHICLES
ABSTRACT

The application of synthetic jets in active flow control in unmanned aerial vehicles has been demonstrated with promising results in the laboratories. While the utility of synthetic jet actuators is effective in altering the apparent aerodynamic shape of the aircraft wing, the designing of such a device is difficult due to the complex physics involved, and compounded by high computational cost in simulation. In order to facilitate the initial design stage of this device, it is imperative adopt a first-order Lumped Element Model to expedite the design process with minimum time and resources. However the current established model only targeted simple conventional designs for single-orifice and single-diaphragm synthetic jet actuators, possessing limited capability of generating sufficiently high flow velocity for unmanned aerial vehicle application.

The work covered in this report seeks to expand the current model to encompass more intricate synthetic jet actuator designs, comprising of multiple independent non-interacting orifices and dual piezoelectric diaphragms operating in-tandem, offering a decent prediction of the frequency response trend at first resonance peak, albeit its limitations as a low-order model. This expanded model has been validated in the later part of the report through a parametric study under quiescent condition, and it has been subsequently verified that the performance of the synthetic jet actuator is enhanced by the inclusion of multiple-orifice and dual-diaphragm features in the design.

Keywords: synthetic jet, lumped element modeling, piezoelectric actuation, multiple-diaphragm, multiple-orifice
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>3</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>4</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td></td>
</tr>
<tr>
<td>CHAPTER ONE</td>
<td>7</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.1 Background</td>
<td>7</td>
</tr>
<tr>
<td>1.2 Objectives</td>
<td>9</td>
</tr>
<tr>
<td>1.3 Report Outline</td>
<td>9</td>
</tr>
<tr>
<td>CHAPTER TWO</td>
<td>11</td>
</tr>
<tr>
<td>REVIEW OF THEORY AND PREVIOUS WORK</td>
<td></td>
</tr>
<tr>
<td>2.1 Synthetic Jets</td>
<td>11</td>
</tr>
<tr>
<td>2.2 Adaptive Virtual Aerosurface</td>
<td>15</td>
</tr>
<tr>
<td>2.3 Modeling of Synthetic Jet Actuator</td>
<td>16</td>
</tr>
<tr>
<td>CHAPTER THREE</td>
<td>23</td>
</tr>
<tr>
<td>LUMPED ELEMENT MODELING</td>
<td></td>
</tr>
<tr>
<td>3.1 Lumped Acoustic Elements</td>
<td>23</td>
</tr>
<tr>
<td>3.1.1 Orifice Linear Acoustic Resistance</td>
<td>23</td>
</tr>
</tbody>
</table>
3.1.2 Orifice Nonlinear Acoustic Resistance 25
3.1.3 Orifice Acoustic Mass 25
3.1.4 Orifice Acoustic Radiation Mass 26
3.1.5 Cavity Acoustic Compliance 29
3.1.6 Diaphragm Acoustic Impedance 30
3.2 Multiple-Diaphragm Actuator Model 34
3.3 Multiple-Orifice Actuator Model 36
3.4 Flow Profile Equations 38

CHAPTER FOUR

EXPERIMENTAL VALIDATION 41
4.1 Design of Actuator Test Rig 41
4.2 Experimental Set-Up 48
4.3 Constant Temperature Hot-Wire Anemometry 51

CHAPTER FIVE 53

ANALYSIS AND DISCUSSION 53
5.1 Conventional Actuator Experimental and LEM Results 53
5.2 Diaphragm Performance 55
5.3 Comparison of Experimental and LEM Results 56
5.4 Design Considerations 58
CHAPTER SIX

CONCLUSIONS AND FUTURE WORK

REFERENCES

APPENDIX A: MATLAB Code for Lumped Element Model

APPENDIX B: CAD Drawings of Configurable Synthetic Jet Actuator

APPENDIX C: Hot-wire Anemometry

APPENDIX D: Damping Coefficient of Piezoelectric Diaphragm

APPENDIX E: Experimental and LEM Results for Various Configurations
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Typical Synthetic Jet Actuator</td>
<td>7</td>
</tr>
<tr>
<td>1.2</td>
<td>Array of Synthetic Jet Actuators Installed on Aircraft Wing</td>
<td>8</td>
</tr>
<tr>
<td>2.1</td>
<td>Acoustic Streaming Criterion</td>
<td>13</td>
</tr>
<tr>
<td>2.2</td>
<td>Synthetic Jet Flow Field</td>
<td>13</td>
</tr>
<tr>
<td>2.3</td>
<td>Piezoelectric Diaphragm Vibration</td>
<td>14</td>
</tr>
<tr>
<td>2.4</td>
<td>Formation of Synthetic Jet at Four Distinct Phases</td>
<td>14</td>
</tr>
<tr>
<td>2.5</td>
<td>Changes in Pressure Distribution of an Airfoil</td>
<td>16</td>
</tr>
<tr>
<td>2.6</td>
<td>Comparison between LEM and TM Models</td>
<td>17</td>
</tr>
<tr>
<td>2.7</td>
<td>Equivalent Circuit Representation of Piezoelectrically-driven SJA</td>
<td>18</td>
</tr>
<tr>
<td>2.8</td>
<td>Schematics of Orifice Geometries</td>
<td>20</td>
</tr>
<tr>
<td>2.9</td>
<td>Schematic of Piezoelectric Diaphragm</td>
<td>22</td>
</tr>
<tr>
<td>3.1</td>
<td>Equivalent Circuit Representation of a Single-Orifice Single-Diaphragm Piezoelectrically-driven SJA</td>
<td>23</td>
</tr>
<tr>
<td>3.2</td>
<td>Cross-Sectional View of Axisymmetric Piezoelectric Composite Plate</td>
<td>30</td>
</tr>
<tr>
<td>3.3</td>
<td>Equivalent Circuit Representation of a Single-Orifice Dual-Diaphragm Piezoelectrically-driven SJA</td>
<td>34</td>
</tr>
<tr>
<td>3.4</td>
<td>Single-Diaphragm Dual-Orifice SJA Equivalent Circuit</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>Representation</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>Dual-Diaphragm Dual-Orifice SJA Equivalent Circuit Representation</td>
<td>36</td>
</tr>
<tr>
<td>3.6</td>
<td>Perforation of an Orifice Plate</td>
<td>38</td>
</tr>
<tr>
<td>4.1</td>
<td>Illustrations of Orifice Plates used for Test Rig</td>
<td>41</td>
</tr>
<tr>
<td>4.2</td>
<td>Spacing between Adjacent Orifices</td>
<td>42</td>
</tr>
<tr>
<td>4.3</td>
<td>SJAs of Different Cavity Compactness Mounted in Wing</td>
<td>44</td>
</tr>
<tr>
<td>4.4</td>
<td>Cross-Sectional Schematic of Configurable SJA</td>
<td>45</td>
</tr>
<tr>
<td>4.5</td>
<td>3D Render of SJA Used in Experiment</td>
<td>45</td>
</tr>
<tr>
<td>4.6</td>
<td>Schematic of Experimental Set-up</td>
<td>49</td>
</tr>
</tbody>
</table>
4.7 Actual Experimental Set-up
4.8 Placement of Hot-wire Probe
4.9 Constant Temperature Anemometry Measurement
4.10 Flow Velocity Calibration
5.1 Case 2 Experimental and LEM Results
5.2 Case 7 Experimental and LEM Results
5.3 Case 9 Experimental and LEM Results
5.4 Comparison of Experimental Results Based on Single-Diaphragm SJA with Different Diaphragms
5.5 Case 12 Experimental and LEM Results
5.6 Case 14 Experimental and LEM Results
5.7 Comparison of Effect of Different Cavity Volumes
5.8 Comparison of Effect of Different Orifice Depths
5.9 Comparison of Effect of Single Circular Orifice with Different Diameters
5.10 Comparison of Effect of Single Slot with Different Lengths
5.11 Single-Slot Single-Diaphragm SJA Experimental Results
5.12 Single-Slot and Double-Slot Single-Diaphragm SJA Experimental Results
5.13 Effect of Different Number of Diaphragms in Operation
5.14 Single-Slot Single-SJA and Dual-Diaphragm SJA Experimental Results
5.15 Conventional and Dual-Slot Dual-Diaphragm SJA Experimental Results
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Given SJA Properties</td>
<td>17</td>
</tr>
<tr>
<td>2.2</td>
<td>List of Lumped Elements</td>
<td>21</td>
</tr>
<tr>
<td>3.1</td>
<td>Matrices [Q], [A], [B], and [D]</td>
<td>32</td>
</tr>
<tr>
<td>4.1</td>
<td>Piezoelectric Disc Technical Specifications</td>
<td>46</td>
</tr>
<tr>
<td>4.2</td>
<td>SJA Test Configurations</td>
<td>48</td>
</tr>
<tr>
<td>5.1</td>
<td>Fixed Geometric Properties of SJAs Simulated in Figure 5.7</td>
<td>59</td>
</tr>
<tr>
<td>5.2</td>
<td>Fixed Geometric Properties of SJAs Simulated in Figure 5.8</td>
<td>61</td>
</tr>
<tr>
<td>5.3</td>
<td>Fixed Geometric Properties of SJAs Simulated in Figure 5.9</td>
<td>62</td>
</tr>
<tr>
<td>5.4</td>
<td>Fixed Geometric Properties of SJAs Simulated in Figure 5.10</td>
<td>63</td>
</tr>
<tr>
<td>5.5</td>
<td>Fixed Geometric Properties of SJAs Simulated in Figure 5.13</td>
<td>65</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\( A_D \) exposed area of diaphragm (m²)

\( a_0 \) orifice characteristic length (m)

\( B \) bulk modulus

\( c_0 \) isentropic speed of sound in air (m/s)

\( C_{ac} \) cavity acoustic compliance (m⁵/N)

\( C_{aD} \) short-circuit acoustic compliance of piezoelectric diaphragm (m⁵/N)

\( C_{eb} \) blocked electrical capacitance of piezoelectric diaphragm (F)

\( C_{EF} \) electrical free capacitance of piezoelectric material

\( d \) largest characteristic length (m)

\( d_A \) Effective acoustic piezoelectric coefficient

\( D_O \) diameter of circular orifice (m)

\( E \) Elastic modulus (Pa)

\( f \) driving frequency (Hz)

\( h \) orifice depth (m)

\( h_c \) mass end correction length (m)

\( h_{eff} \) effective neck length (m)

\( h_o \) characteristic length of the orifice (m)

\( k \) wave number (m⁻¹)

\( K \) non-dimensional loss coefficient

\( K_c \) orifice conductivity (m)
$L$  dimensionless stroke length

$L_{O}$  slot length (m)

$L_s$  stroke length (m)

$M_{aD}$  short-circuit acoustic mass of piezoelectric diaphragm (kg/m$^4$)

$M_{aN}$  orifice neck acoustic mass (kg/m$^4$)

$M_{aRad}$  orifice acoustic radiation mass (kg/m$^4$)

$P$  sinusoidal oscillating pressure

$Q$  total fluid flow (m$^3$/s)

$Q_c$  cavity fluid (m$^3$/s)

$Q_{out}$  output fluid flow (m$^3$/s)

$R_{aD}$  acoustic resistance of piezoelectric diaphragm (kg/s·m$^4$)

$R_{aDRad}$  acoustic radiation resistance of piezoelectric diaphragm (kg/s·m$^4$)

$R_{aN}$  orifice neck linear acoustic resistance (kg/s·m$^4$)

$R_{aO}$  orifice nonlinear acoustic resistance (kg/s·m$^4$)

$Re$  effective radius of piezoelectric diaphragm (m)

$Re$  Reynolds number

$R_{loss}$  piezoceramic dielectric loss (kg/s·m$^4$)

$s$  Laplace variable

$S_n$  orifice cross-sectional area (m$^2$)

$Sr$  Strouhal number

$St$  Stokes number

$V_{ac}$  sinusoidal A.C. voltage input (V$_{pp}$)
\[ U \] velocity of flow (m/s)
\[ U_{\text{max}} \] maximum velocity (m/s)
\[ \bar{U} \] mean velocity (m/s)
\[ \nu \] axial velocity component (m/s)
\[ W_O \] slot width (m)
\[ x_a \] perforate constant
\[ Z_{aD} \] acoustic impedance of diaphragm (kg/s·m\(^4\))
\[ Z_{aDRad} \] acoustic radiation impedance of diaphragm (kg/s·m\(^4\))
\[ Z_{aO} \] acoustic impedance of orifice (kg/s·m\(^4\))
\[ \phi \] ideal transformer turns ratio
\[ \omega \] angular frequency (rad/s)
\[ \rho_0 \] fluid density (kg/m\(^3\))
\[ \rho_D \] piezoelectric diaphragm density (kg/m\(^3\))
\[ \lambda \] acoustic wavelength (m)
\[ \nu \] kinematic viscosity of fluid (m\(^2\)/s)
\[ \mu \] dynamic viscosity of fluid (Pa·s)
\[ \zeta \] damping coefficient of diaphragm
\[ \epsilon \] eccentricity of an elliptic orifice
\[ \epsilon \] volume dilation
\[ \sigma_o \] perforate porosity
\[ \Delta P \] static pressure difference between cavity and ambient pressure (Pa)
\[ \Delta V \] fluid volume displaced by diaphragm (m\(^3\))
CHAPTER 1: INTRODUCTION

1.1 Background

The Synthetic Jet Actuator (SJA) has been subjected to extensive researched in the past decades for its potential utility in flow control application. The SJA is a miniscule electrically-powered fluid pump, which operates in a similar manner to the biological respiratory system. The surrounding air is drawn into the cavity, analogous to the lungs, and exhaled through the opening in this Zero-Net Mass-Flux (ZNMF) device. A flexible wall or diaphragm, located on the side of the device, through a series of expansion and contraction motion, generates the effect of inhalation and exhalation respectively. This ‘breathing’ action is consecutively reiterated several times within a short time frame which consequently, creates pulses of air jets known as Synthetic Jets (SJs), in essence comprises of numerous vortices similar to smoke rings. A 2D schematic of a simple SJA is as shown in Figure 1.1.

![Figure 1.1 Typical Synthetic Jet Actuator](image)

The SJA, has seen several applications in flow control[1], thermal management[2], fluid mixing enhancement[3], etc. due to its simplicity and versatility. It has been proven in the laboratory that the SJs possess the latent capability to delay flow separation which principally substantiate the prospect of decreasing aerodynamic drag and augmenting aircraft lift especially during the takeoff and landing phase. In aerospace application, there is potential for improved aircraft performance and safety by utilizing properly designed
and strategically located arrays of SJAs to (i) delay stall and enhance aircraft controllability at high angle of attack, and to (ii) provide cost-effective actuation redundancy especially in the case of control surface malfunction \[^4\]. This motivates the interest to understand the complex dynamics involved in SJAs and design an optimal SJA to be implemented in small Unmanned Aerial Vehicles (UAVs) as an active flow controller based on piezoelectric composite technology. An array of SJAs installed on an aircraft wing is as illustrated in Figure 1.2.

![Figure 1.2 Array of Synthetic Jet Actuators Installed on Aircraft Wing][4]

Several computational means have been adopted to model the SJA to facilitate its design process, but these methods proved to be time-consuming and have relatively high computational cost due to the vastly complex physics involved, which is impractical for the initial stage of SJA design. Additionally, details concerning the operation and design of the SJA were lacking in some of these models. To facilitate the design of a SJA, low-order modeling is often required to circumvent intensive and time-consuming experiments and Computational Fluid Dynamics (CFD) simulations. Two low-cost methods are available to provide a convenient and intuitive insight into the dynamics of SJA design, the first being the quasi-static first-order Lumped Element Modeling (LEM) and the second is the Transfer Matrix (TM) modeling.
1.2 Objectives

This FYP-URECA project aims to design, analyze, and test small-scale SJA for application on small UAVs, by espousing a low-order LEM model convenient for initial design stages, specifically to predict the SJA frequency response for multiple independent straight circular and slot orifices, and dual piezoelectric composite diaphragms in operation. Despite the fact that LEM is predicated on a fundamental assumption that is circumvented by the TM model, numerical solution based on LEM is simpler and in the case of implementing SJA on UAVs, the size of the SJA is inherently small such that the constraint imposed by the assumption is trivial. Hence, the selection of LEM as the model for this research is sufficient for a reasonable prediction of the SJA performance.

The scope of the research involved encompasses (i) the identification of key SJA parameters and design features namely, similar multiple-diaphragm and similar multiple-orifice, relevant to the optimization of the resulting flow velocity at Helmholtz resonance, (ii) the extension of the existing LEM model to account for new design features introduced, (iii) design and fabrication of configurable SJA test rig for parametric study and validation of the extended LEM model with design constraints based on an actual UAV wing, and (iv) the selection of suitable SJA design appropriate for installation on an UAV wing.

1.3 Report Outline

This report is structured and segregated into various chapters to discuss the various aspects of the research. The next chapter describes the dynamics of synthetic jets in detail, including the formation of synthetic jets and its range of ameliorating effect to the surrounding flow relevant to the aerospace application which institutes an alternative means of flow control through Adaptive Virtual Aerosurface (AVIA), in addition to the discussion of the existing LEM model of a piezoelectric SJA and its existing formulation of the LEM parameters. Insights to the various LEM parameters can be found in Chapter Three of this report, which covers the analytic derivations of these parameters and the estimations of its nonlinear effects, as well as the inclusion of similar multiple-diaphragm
and multiple-orifice equivalent impedance-type electroacoustic circuit representation and equations as part of the new design features introduced.

Chapter Four discusses the design of the configurable SJA test rig for LEM validation by considering the various implications involved such as the placement of the adjacent orifices for multiple-orifice cases, and also the conduct of the experiment based on hot-wire anemometry. LEM results are compared to the experimental data in Chapter Five and the limitations of the model is further elaborated. Chapter Five also analyses the various dimensional parameters and a number of design considerations are suggested for improving a conventional SJA performance on top of the proposed SJA design recommended for implementation on the prescribed UAV wing.

Finally, this report will be concluded in Chapter Six, with the recommendation of possible future works that can be conducted to further develop the LEM model for a multiple-orifice multiple-diaphragm SJA.
CHAPTER 2: REVIEW OF THEORY AND PREVIOUS WORK

2.1 Synthetic Jets

A synthetic jet flow is produced through movement of surrounding fluid back and forth through an orifice. The periodic entrainment and ejection of the fluid is induced by the oscillatory motion of a diaphragm.

In aircrafts, boundary flow separation posed a substantial problem in aerodynamics and performance. The occurrence of flow separation is due to the presence of adverse pressure gradient that decelerates the speed of the boundary layer relative to the object of concern e.g. aircraft wing, to zero. Subsequently, the fluid flow detaches itself from the object surface and exhibits itself in the form of vortices and as a result, aerodynamic penalties due to increased drag ensued. Also, for internal flow systems, undesirable effects pertaining to higher flow losses and stall-type phenomena such as compressor surge will transpire\[^5\]. Therefore it is essential to adopt flow separation control techniques to diminish or even mitigate flow separation, which unfortunately introduces minor parasitic drag, higher energy requirement, and necessary installation of devices.

Generally, there are four main types of separation control\[^6\]: (i) tangential blowing such as leading edge slats and slotted flaps to energize the low-momentum region near the wall, (ii) wall suction for removal of low-momentum region, (iii) using vortex generators to boost convective transport of momentum from freestream to the wall region, and (iv) forced excitation upstream of the point of separation, also known as ‘dynamic forcing’, by manipulating the separated shear layer’s inherent instability due to perturbations. Conventionally, the problem associated with tangential blowing and wall suction, is that complex internal piping is required and the accrued cost of generating pressure source for these approaches. As for vortex generators, despite their simplicity, higher parasitic drag is resulted and these passive vortex generators are also incapable of handling leading edge separation. For dynamic forcing, the periodic excitation of the flow with instances such as
vibrating flap or oscillating slot flow, the shear layer roll up of vortices which generates vortex structures over the surface downstream increases flow turning.

A mean flow stream that emanates through the orifice from high levels of diaphragm excitation and the resulting mean streamlines formed a closed recirculation, and such phenomena is also known as ‘acoustic jet streaming’. Shown in Figure 2.1, four circulation and turbulence regimes can be seen around an orifice plate that is acoustically-excited[7]. The dashed line denotes the point where particle displacement in the orifice is equivalent to the orifice length. In region 1 where low excitation levels are experienced, fluid particles move across the orifice without mingling with the external fluid particles, i.e. particles outside the orifice. Small acoustic energy losses due to acoustic radiation is lost to the surroundings. In region 3, close to and above the dashed line, substantial turbulence is developed around the orifice which causes some acoustic energy to be translated into mean fluid motion. For high excitation levels in region 4, the fluid particles displace out of the orifice and this accounts for sufficient amount of time for fluid particles to roll up into vortex rings and convect away from the orifice region. Relatively long residence time for fluid particles to be exposed to the external shear enables Kelvin-Helmholtz instability to develop and thus generating well defined vortex structures that propagate away from the orifice via induced-vortex motion, effectively forming a mean flow stream or SJ, as illustrated in Figure 2.2. Opposed to continuous jets, SJs spread in a different manner due to near-field dissimilarities, resulting in a greater growth in mass flow rate[8].

A typical SJA comprises of three main components, namely, the cavity, diaphragm, and the orifice. The diaphragm manifests itself in the form of either an electromagnetic driver in the case of plasma actuator, a piezoelectric driver which is used in this project, or a mechanical driver such as shaker. Once actuated, the diaphragm deflects in accordance to the driving frequency which in turn produces a fluctuating pressure difference between the cavity and the external flow.
Piezoelectric composite diaphragm is preferred over piston-driven actuators for UAV application due to its compactness and light-weight, despite the latter being more reliable and powerful. Two main operating phase of the unimorph piezoelectric diaphragm deflection is identified as shown in Figure 2.3, namely the upwards deflection or contraction phase, and the downwards deflection or expansion phase.

For a typical SJA with a singular orifice, during the contraction phase, the cavity pressure surged and this precipitated into an expulsion of fluid flow out of the cavity volume. As flow separates from the orifice edge, a vortex ring is formed. With sufficiently large initial impulse transmitted, the vortex ring is carried downstream by its own velocity, leaving a trailing shear layer. In contrast, during the expansion phase, the external ambient flow is entrained back into the cavity. After suction, a vortex ring is formed on the internal side of the orifice. Figure 2.4 depicts the four distinct phases of SJ operation\(^9\). By repeating the contraction-expansion cycle consecutively, a chain of vortex structures is created. This operating cycle highlighted the ability of the SJA to transport momentum to the external flow to re-energize the flow without any net mass input.

The SJA can be operated under three distinct regimes for aircrafts: (i) at the wing surfaces, the SJA is driven at low frequency at high angles of attack to delay flow separation which is useful for preventing stall\(^10\), (ii) in fully-attached flows, the SJA is driven at high frequency with adequate jet velocity to create regions of recirculation flow that can alter
the aerodynamic shape of the aircraft and successively adjust the load distribution\textsuperscript{[11]}, and (iii) SJA is used to reduce the acoustic loads which propounds that SJs are endowed with the potential to cancel Tollmien-Schlichting waves essential for laminar flow control\textsuperscript{[12]}. For near-wall turbulent boundary layer control, however, low Reynolds number and high Strouhal number is preferred where velocities are of the order of the turbulent friction velocity, with the intention of preventing the SJ from blowing through the boundary layer\textsuperscript{[13]}. It is determined that high Reynolds number ensures that the vortex rings generated by the orifice can move farther away from the orifice as they are less affected by the entrainment phase of the SJA. Strouhal number governs the unsteadiness and directivity of the SJ as a fluid element traverse the orifice at the actuation frequency\textsuperscript{[14]}.
2.2 Adaptive Virtual Aerosurface

Conventionally, control surfaces are applied to attain aerodynamic control of aerial vehicles. However, AVIA provides an alternative method by producing an unsteady momentum flux on an aerodynamic surface. The corresponding flow field and surface pressure distribution can thus be altered in the absence of physical changes or control surfaces movement i.e. flow interactions used for aerodynamic control instead of traditional physical structures\textsuperscript{15}. In UAV application, the utilization of SJs has shown tremendous potential in shaping the AVIA.

While several passive methods for flow separation control exists, such as vortex generator (VG), these approaches bring about an undesirable effect of slightly reduced performances at lower angle of attack. On the other hand, SJs have been proved to be able to transform the apparent aerodynamic shape of a vehicle body with reduced actuation frequencies of $O(1\text{--}100)$\textsuperscript{16}.

In the dynamics of flow separation control, the initial reattachment of the boundary layer is dependent on the nature of the impulse or initial transient, as opposed to non-dimensional frequency which is essential in defining the conditions when detachment occurs. While the introduction of perturbations at the appropriate frequency relative to the time scales of the flow serves to ensure that the flow is attached indefinitely, an impulse is sufficient in reattaching the flow momentarily. Hence, by triggering in the most intricate moment, flow separation can be prevented\textsuperscript{17}. The utilization of SJAs on UAVs has been demonstrated with significant pitch and roll authority at angles of attacks for which the wing is partially stalled\textsuperscript{18}[19].

An example of a change in pressure distributions on an airfoil just by activating SJAs\textsuperscript{20} can be seen in Figure 2.5. The creation of recirculation region close to the orifice causes an apparent new boundary to be formed with respect to the local streamlines, which substantially alters the flow field surrounding the airfoil and hence influencing the aerodynamic characteristics. Consequently, the mechanical complexity and weight arising
from conventional control surfaces can be circumvented by using SJAs to modify the pressure distribution signature around the airfoil.

![Figure 2.5 Changes in Pressure Distribution of an Airfoil](image)

### 2.3 Modeling of Synthetic Jet Actuator

The LEM analysis is common performed on coupled-domain transducer system\[21\]; thus the SJA which is primarily an electrically, mechanically and acoustically coupled system, is compatible to the employment of LEM, and it was initially modeled on a speaker-driven SJA\[6\]. In LEM, the behavior of a SJA is determined from its geometry and material properties of the diaphragm. Key performance parameters, i.e. resonance frequency and volumetric flow output, are obtained from the numerical solution. This model provides results in good agreement with the experiment in the comparison of the predicted actuator performance with the actual experimental data\[22\].

However, LEM is fundamentally constrained by its underlying assumption on the compactness of the device. The maximum length of any of the SJA’s dimension should
be much lesser than its acoustic wavelength, hence there is a limit imposed on the maximum achievable SJA dimension. If this assumption holds true, the partial differential equations governing the distributed system can be conveniently lumped together into a set of coupled ordinary differential equations. The new TM method, on the other hand, is capable of circumventing the limitations imposed by LEM\cite{23}, albeit limited to frequencies between direct current (DC) and just beyond the diaphragm’s fundamental frequency. Figure 2.6 compares the LEM and TM results with the experimental data for a piezoelectric SJA, with properties specified in Table 2.1.

![Figure 2.6 Comparison between LEM and TM Models\cite{23}] (image)

Despite the dimensional limitations imposed by the LEM, the model provides a reliable and simplistic means of predicting the SJA output flow velocity given the required inputs: (i) material properties of the diaphragm, (ii) SJA geometry, and (iii) operating conditions. Therefore, suitable SJA geometry and operating conditions can be selected during the design process. The absence of grazing flow is assumed in order to simplify the model.

**Table 2.1 Given SJA Properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity (Cylindrical) Depth</td>
<td>0.67mm</td>
</tr>
<tr>
<td>Piezoelectric Disc Material</td>
<td>PZT-5A</td>
</tr>
<tr>
<td>Shim Material</td>
<td>Brass</td>
</tr>
<tr>
<td>Piezoelectric Disc Radius</td>
<td>10.05mm</td>
</tr>
<tr>
<td>Shim Radius (Clamped)</td>
<td>13.05mm</td>
</tr>
<tr>
<td>Orifice Type</td>
<td>Circular</td>
</tr>
<tr>
<td>Orifice Depth</td>
<td>2.5mm</td>
</tr>
<tr>
<td>Orifice Radius</td>
<td>0.55mm</td>
</tr>
</tbody>
</table>
The basic underlying assumption behind the effectiveness of LEM is predicated on the premise that the largest characteristic length of the SJA must be much smaller relative to the acoustic wavelength of concern, i.e. the LEM model is only accurate for frequency range not exceeding the prescribed upper frequency bound where the characteristic length is no longer considerably small.

Due to the complex physics involved, it is necessary to partition the consideration of the operating SJA into two domains – electrical-mechanical domain and the acoustic domain. Figure 2.7 shows the equivalent circuit representation of a piezoelectrically-driven SJA. The piezoelectric diaphragm consists of a clamped axisymmetric metal plate with a piezoceramic disc centrally-mounted on it. In the LEM model presented, an impedance-type acoustic circuit is implemented in the acoustic domain, where the pressure is the effort variable while volume flow rate is the flow variable[^25].

![Figure 2.7 Equivalent Circuit Representation of Piezoelectrically-driven SJA][24]

In the electrical domain, $V_{ac}$ is the AC voltage source input, typically expressed in its phasor form, and in this report it is expressed as a peak-to-peak voltage. $C_{eb}$ is the blocked electrical capacitance of the piezoelectric diaphragm, and an additional $R_{loss}$ can be added in series or parallel to $C_{eb}$ to represent the piezoceramic dielectric loss.

As for the lumped elements in the acoustic domain, $C_{aD}$ is the short-circuit acoustic compliance of the piezoceramic composite diaphragm, and $M_{aD}$ is the short-circuit acoustic mass of the piezoceramic composite diaphragm. $R_{aD}$ is the piezoelectric
diaphragm’s acoustic resistance associated with the structural damping, and \( R_{aDRad} \) may also be supplemented to the diaphragm impedance if the external-side of the piezoelectric diaphragm is radiating into an open medium, with the acoustic radiation impedance for the diaphragm\[^{[26]}\] expressed as

\[
Z_{aDRad} = R_{aDRad} + jM_{aDRad} = \rho_0 c_0 A_D \left( 1 - \frac{J_1(2kR_e)}{kR_e} + j \frac{H_1(2kR_e)}{kR_e} \right)
\]  
(1)

where \( A_D \) is the exposed surface area of the diaphragm, \( R_e \) is the effective radius of the exposed diaphragm, \( J_1 \) is a first-order Bessel function of the first kind, \( H_1 \) is a first-order Struve function, and \( k \) is the wave number given as

\[
k = \frac{2\pi f}{c_0}
\]  
(2)

As for the cavity, its potential energy storage capacity is represented by \( C_{aC} \), the cavity acoustic compliance. For the orifice portion, \( R_{aN} \) represents the linear acoustic resistance due to viscous losses, and \( M_{aN} \) represents the acoustic mass of the fluid in the orifice neck. \( R_{aO} \) is the nonlinear acoustic resistance attributed to vorticity in the orifice discharge and \( M_{aRad} \) is the acoustic radiation mass.

A conversion from the electrical domain to the acoustic domain is accounted for using an ideal transformer with turns ratio \( \phi \), which is also known as the transduction coefficient. The diaphragm motion can be seen as a compression or expansion action on the cavity fluid to eject or ingest the fluid through the orifice, hence the total fluid flow can be conveniently expressed as

\[
Q = Q_C + Q_{out}
\]  
(3)

where \( Q_C \) embodies the cavity fluid, while \( Q_{out} \) is the fluid flow out of the orifice.
For $s = j\omega$, the Laplace transformed expression for the frequency response function is written as

$$\frac{Q_{out}(s)}{V_{ac}(s)} = \frac{\phi C_{aD}s}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + 1}$$ (4)

where

$$a_1 = (R_{aN} + R_{aO})(C_{aD} + C_{aC})$$

$$a_2 = (M_{aRad} + M_{aN})(C_{ab} + C_{ac}) + M_{aD}C_{aD}$$

$$a_3 = C_{ac}M_{aD}C_{ab}(R_{aN} + R_{aO})$$

$$a_4 = C_{ac}M_{aD}C_{ab}(M_{aRad} + M_{aN})$$

The lumped elements based on Gallas\cite{24}, Prasad\cite{27}, and Tang\cite{28}, are as compiled in Table 2.2, for various orifices and diaphragm types. For an electrodynamic actuator driven by a magnetic coil, Sawant’s work\cite{29} may be referenced. Schematics of the orifice geometries and diaphragm specified in the literature are as illustrated in Figure 2.8 and 2.9 for orifice and piezoelectric diaphragm respectively.

![Figure 2.8 Schematics of Orifice Geometries](image)
Table 2.2 List of Lumped Elements[24][27][28]

<table>
<thead>
<tr>
<th>Lumped Elements</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Orifice Linear Acoustic Resistance (Neck Viscous Losses)</strong></td>
<td>( R_{aN} = \frac{12\rho_0 v h}{\pi D_0^4} )</td>
</tr>
<tr>
<td><strong>Orifice Acoustic Mass (Neck)</strong></td>
<td>( M_{aN} = \frac{16\rho_0 h}{3\pi D_0^2} )</td>
</tr>
<tr>
<td><strong>Orifice Nonlinear Acoustic Resistance (Discharge)</strong></td>
<td>( R_{aO} = 8K\rho_0 \frac{Q_{out}}{\pi^2 D_0^4} )</td>
</tr>
<tr>
<td><strong>Orifice Acoustic Radiation Resistance</strong></td>
<td>( R_{aRad} = 0.159\frac{\rho_0 a^2}{a} )</td>
</tr>
<tr>
<td><strong>Orifice Acoustic Radiation Mass</strong></td>
<td>( M_{aRad} = \frac{16\rho_0}{3\pi^2 D_0} )</td>
</tr>
<tr>
<td><strong>Orifice Linear Acoustic Resistance (Neck Viscous Losses)</strong></td>
<td>( R_{aN} = \frac{3\rho_0 v h}{2 \left( \frac{W_0}{2} \right)^3 L_0} )</td>
</tr>
<tr>
<td><strong>Orifice Acoustic Mass (Neck)</strong></td>
<td>( M_{aN} = \frac{6\rho_0 h}{5W_0 L_0} )</td>
</tr>
<tr>
<td><strong>Orifice Nonlinear Acoustic Resistance (Discharge)</strong></td>
<td>( R_{aO} = \frac{1}{2} K\rho_0 \frac{Q_{out}}{W_0^2 L_0^2} )</td>
</tr>
<tr>
<td><strong>Orifice Acoustic Radiation Mass</strong></td>
<td>( M_{aRad} = \frac{\rho_0}{\pi L_0} \left( \frac{L_0}{W_0} \log \frac{2}{W_0} + \frac{1}{2(1 - \omega^2 L_0^2)} \right) )</td>
</tr>
<tr>
<td><strong>Cavity Acoustic Compliance</strong></td>
<td>( C_{aC} = \frac{V}{\rho_0 c_0} )</td>
</tr>
<tr>
<td><strong>Diaphragm Acoustic Mass</strong></td>
<td>( M_{aD} = \frac{2\pi \rho_0}{A V} \int_0^{R_e} (w(r))^2 r dr )</td>
</tr>
<tr>
<td><strong>Diaphragm Acoustic Compliance</strong></td>
<td>( C_{aD} = \frac{2\pi}{P} \left[ b_1^{(1)} \left( -R_1^4 \right) + b_1^{(2)} \left( \frac{(R_1^4 - R_2^4)^2}{8} + \frac{PR_1^6}{192D_{11}^{(1)}} \right) \right] )</td>
</tr>
<tr>
<td><strong>Diaphragm Acoustic Resistance</strong></td>
<td>( R_{aD} = 2\zeta \frac{M_{aD}}{C_{aD}} )</td>
</tr>
<tr>
<td><strong>Damping Coefficient</strong></td>
<td>( \frac{1}{2} R_{aN} \sqrt{C_{aC} \frac{M_{aN}}{M_{aO}}} )</td>
</tr>
</tbody>
</table>
Figure 2.9 Schematic of Piezoelectric Diaphragm
CHAPTER 3: LUMPED ELEMENT MODELING

3.1 Lumped Acoustic Elements

Since the fundamental acoustic wavelength is much larger than the characteristic length of the SJA, the behavior of large wavelength waves within small edifices can be approximated by assuming pressure uniformity within the volume of interest\textsuperscript{30}. Hence, various components of the SJA can in turn be analyzed in terms of ideal lumped acoustic elements, which can be estimated experimentally, analytically or numerically. As such, spatial considerations within the lumped elements can be neglected.

In the absence of a cross-flow, the lumped element model of a basic single-orifice single-diaphragm piezoelectrically-driven SJA can be represented as shown in Figure 3.1.

![Figure 3.1 Equivalent Circuit Representation of a Single-Orifice Single-Diaphragm Piezoelectrically-Driven SJA](image)

In the acoustic domain of the representative circuit diagram, $Z_{\text{aD}}$ denotes the acoustic impedance of the diaphragm while $Z_{\text{aO}}$ denotes the acoustic impedance of the orifice. Acoustic impedance is defined as the ratio of sound pressure to the volume flow.

3.1.1 Orifice Linear Acoustic Resistance

For viscous movement of air through a capillary tube, dissipative losses can be represented by the acoustic resistance. Two acoustic resistances, namely linear acoustic resistance and non-linear acoustic resistance, will be considered to account for viscous loss and vortex formation for oscillating flows respectively.
The solution of viscous loss for low Reynolds number steady flow in a circular pipe is obtained using Hagen-Poiseuille equations. In a laminar regime fully-developed flow, the velocity distribution is

\[ U(r) = \frac{1}{4\mu} \left( \frac{D_o^2}{4} - r^2 \right) \left( \frac{dP}{dx} \right) \]  

(5)

where \( \mu \) is the dynamic viscosity of the fluid.

Integrating (5), the volume flow can be expressed as

\[ Q = \frac{\pi D_o^4}{128\mu} \left( \frac{dP}{dx} \right) \]  

(6)

As the flow velocity-dependent resistance can be empirically obtained as

\[ R_{aN} = \frac{\Delta P}{\bar{U} S_n} = \frac{\Delta P}{Q} \]  

(7)

where \( \Delta P \) is the static pressure difference between cavity and the ambient pressure, and \( \bar{U} \) is the mean velocity.

The linear acoustic resistance of a circular orifice due to viscous losses is thus

\[ R_{aN} = \frac{128\mu h}{\pi D_o^4} \]  

(8)

Assuming a laminar fully-developed flow through a 2D channel, for small \( \omega \), where \( x \) is the relative span-wise position from the midpoint, the flow profile for a steady flow is

\[ U(x) = \frac{\Delta P}{2\mu h} \left( \frac{W_o^2}{4} - x^2 \right) \]  

(9)

By assuming low frequencies during operation, i.e. \( kd = \omega d/c_0 \ll 1 \), the volume velocity can be represented in (10)

\[ Q = \int_{-0.5W_o}^{0.5W_o} \int_{0}^{L} U(x) \, dx \, dz = \frac{L_o \Delta P}{2\mu h} \left( \frac{W_o}{2} \right)^2 \cdot \left[ \frac{4}{3} \left( \frac{W_o}{2} \right) \right] \]  

(10)
Substituting (10) into (7), the linear resistance of a 2D slot is

\[ R_{aN} = \frac{3\mu h}{2L_0 \left(\frac{W_0}{2}\right)^3} \]  

(11)

### 3.1.2 Orifice Nonlinear Acoustic Resistance

With regards to the dump-loss term for an orifice discharge, the nonlinear acoustic resistance can be approximated by modeling the orifice as a generalized Bernoulli flow meter\[^{31}\]

\[ R_{aO} = \frac{1}{2} K \rho_0 \frac{Q_{out}}{S_n} \]  

(12)

where \( K \) is a non-dimensional loss coefficient which is dependent on the geometry of the orifice, operating frequency, and Reynolds number. However, \( K \) is assumed to be unity with reasonable accuracy in the model presented.

Thus, the discharge acoustic resistance for a circular orifice and a slot is given in equation (13) and (14) respectively

\[ R_{aO} = 8K \rho_0 \frac{Q_{out}}{\pi^2 D_0^4} \]  

(13)

\[ R_{aO} = \frac{1}{2} K \rho_0 \frac{Q_{out}}{D_0^2 B_0^2} \]  

(14)

### 3.1.3 Orifice Acoustic Mass

Acoustic mass, or inertance, in essence, is a mass of air accelerated by a net force, which displaces the air without compressing it substantially. In low frequency limit, i.e. \( \lambda r \ll 1 \), for a short open tube, the air will be displaced by an equal amount on both sides of the opening under an external pressure on one of its ends.
The orifice linear acoustic mass for a fully-developed pipe flow is reflected as an unsteady inertia effect which can be characterized as an inertial mass in the orifice. As the kinetic energy of the flow can be expressed as

\[ K.E. = \frac{1}{2} M_{AN} Q^2 \]  

(15)

For a Poiseuille flow, the circular orifice acoustic mass can be derived as

\[ M_{AN} = \frac{4 \rho_0 h}{3 \pi \left( \frac{D_o}{2} \right)^2} \]  

(16)

and for a slot orifice, (15) can be rewritten as

\[ K.E. = \frac{1}{2} Q^2 \rho_0 h \frac{3}{5 L_o \left( \frac{W_o}{2} \right)} \]  

(17)

and thus the orifice acoustic mass for a slot is

\[ M_{AN} = \frac{3 \rho_0 h}{5 L_o \left( \frac{W_o}{2} \right)} \]  

(18)

3.1.4 Orifice Acoustic Radiation Mass

The device orifice is perceived as a thin piston of air in an infinite baffle. Given an acoustic excitation, this piston column will radiate sound waves back towards its source and such effect manifests itself in the form of acoustic radiation impedance in the acoustic circuit. Given that the orifice cross-sectional area for a uniform neck is \( S_n \), the acoustic response of this component is subjected to a sinusoidal oscillating pressure \( P(\omega) \).

From Newton’s second law,

\[ P(\omega) A = \rho_0 h S_n \frac{dU(\omega)}{dt} \]  

(19)

where \( P(\omega) \) and the fluctuating velocity \( U(\omega) \) can be expressed respectively as
\[ P(\omega) = P_f e^{j\omega t} \]  
\[ U(\omega) = U_f e^{j\omega t} \]

The volume velocity response to the oscillating pressure varies in a sinusoidal manner with frequency \( \omega \), thus (19) can be rewritten as

\[ P(\omega) = j\omega \frac{\rho_0 h}{s_n} Q(\omega) \]  
\[ (22) \]

where \( Q(\omega) = S_n U(\omega) \) is the sinusoidal volume velocity.

By performing a Laplace transform on a completely reactive acoustic inertance with frequency variable \( s = j\omega \), the acoustic mass of the orifice for negligible initial volume velocity is given by

\[ Z(s) = \frac{P(s)}{Q(s)} = \frac{\rho_0 h}{S_n} s \]

\[ (23) \]

Since the acoustic wavelength is considerably longer than the orifice dimensions, the radiation impedance term can be neglected. Taking into consideration the mass end correction, the effective neck length of the orifice is given as

\[ h_{eff} = h + h_c \]

\[ (24) \]

where \( h_c \) is the mass end correction length, which is unambiguously the mass inertial end correction for radiation impedance.

According to Ingard\[7\], for a circular orifice or slot in a flanged plate with \( \xi < 0.4 \), the end correction can be approximated as

\[ h_c \equiv 0.48\sqrt{S_n}(1 - 1.25\xi) \]

\[ (25) \]

where \( \xi \) for circular orifice and slot is written as (26) and (27) respectively

\[ \xi = \frac{D_0}{D_{plate}} \]

\[ (26) \]
\[ \xi = \frac{W_o}{W_{\text{plate}}} \]  

(27)

For a small orifice at the flanged end in an infinite baffle, \( \xi \approx 0 \). By applying this condition to (23), the correction can be approximated as

\[ h_c = \frac{4D_0}{3\pi} \]  

(28)

Therefore, the acoustic radiation mass of a circular orifice adopted by Gallas\textsuperscript{[24]} in his LEM model is given as

\[ M_{\text{arad}} = \frac{4D_0 \rho_0}{3\pi S_n} = \frac{16\rho_0}{3\pi^2 D_O} \]  

(29)

Also, it is noted that for an elliptic orifice, similar approach is used to estimate for the acoustic radiation mass, except that the value of \( S_n \) used in (29) is approximately based on equivalent circle area for its elliptic area\textsuperscript{[32]}.

The mass end correction for an elliptic orifice can also be presented\textsuperscript{[33]} as

\[ h_c = \frac{S_n}{K_C} \]  

(30)

where \( K_C \) is referred to as orifice conductivity. The orifice conductivity of an elliptical orifice derived by Rayleigh\textsuperscript{[34]} is

\[ K_C = 2 \frac{S_n}{\sqrt{\pi}} \left( 1 + \frac{\epsilon^4}{64} + \frac{\epsilon^4}{64} + \cdots \right) \]  

(31)

where \( \epsilon \) is the eccentricity of the elliptical orifice.

According to Meissner\textsuperscript{[35]}, the acoustic radiation mass for a slot is modeled for \( kd < 1 \) as a rectangular piston in an infinite baffle predicated on the assumption that the slot is on a plate much larger dimensionally than the slot size.
Hence the reactance corresponding to the acoustic radiation is

\[ M_{\text{aRad}} = \frac{\rho}{\pi B_0} \left( \frac{B_0}{D_0 \log \left( \frac{2 B_0}{D_0} \right)} + \frac{1}{2 \left( 1 - \frac{\omega^2 B_0^2}{6a^2} \right)} \right) \]  

(32)

3.1.5 Cavity Acoustic Compliance

In the low frequency limit, the application of an external pressure to compress the enclosed volume will result in a spring-like behavior due to the air’s elasticity\cite{36}. Volume dilation is the ratio of volume change to the original internal volume, given as

\[ \epsilon = \frac{\Delta V}{V} \]  

(33)

In view of small dilation, Hooke’s Law for fluid can be used as an accurate approximation to the relationship between the change in pressure, the resulting dilation, and bulk modulus \( B \),

\[ P = -B \epsilon \]  

(34)

As the fluid of concern can be considered as nearly adiabatic, the bulk modulus can be written as

\[ B = \rho_0 c_0^2 \]  

(35)

The change in cavity volume can be re-expressed as

\[ \Delta V = \int Q dt = \frac{Q}{j\omega} \]  

(36)

Hence the Laplace transformed acoustic impedance of the cavity is

\[ Z(s) = \left( \frac{\rho c_0^2}{V} \right) \frac{1}{s} \]  

(37)
The cavity acoustic compliance, or the acoustic spring, can be written as

\[ C_{ac} = \frac{V}{\rho_0 c_0^2} \]  

(38)

### 3.1.6 Diaphragm Acoustic Impedance

The parameters for the piezoelectric unimorph can be modeled using an axisymmetric composite plate two-port electroacoustic network model\(^{[27]}\). Figure 3.2 shows the cross-sectional view of the composite plate.

![Figure 3.2 Cross-Sectional View of Axisymmetric Piezoelectric Composite Plate\(^{[27]}\)](image)

For a low-dimensional piezoelectric electromechanical coupling, the coupling equations can be written in the matric form

\[
\begin{bmatrix}
\Delta V \\
q
\end{bmatrix} =
\begin{bmatrix}
C_{ad} & d_A \\
d_A & C_{EF}
\end{bmatrix}
\begin{bmatrix}
P \\
V
\end{bmatrix}
\]  

(39)

The volume displaced can be obtained by integrating the transverse displacement encompassing the entire composite plate

\[ \Delta V = \int_0^{R_2} 2\pi rw(r)dr \]  

(40)

The electrical free capacitance of the piezoelectric material in the absence of pressure loading is

\[ C_{EF} = \frac{\varepsilon_r \varepsilon_0 \pi R_1^2}{h_p} \]  

(41)
The piezoelectric diaphragm short-circuit acoustic compliance and the effective acoustic piezoelectric coefficient is defined in (42) and (43) respectively

\[ C_{aD} = \frac{\Delta V}{P} \bigg|_{V=0} \quad (42) \]

\[ d_A = \frac{\Delta V}{V} \bigg|_{p=0} \quad (43) \]

Consequently, by application of (41) into (42) and (43), the updated equations are (44) and (45) respectively

\[ C_{aD} = \frac{\int_0^{R_2} 2\pi r w(r) \bigg|_{V=0} dr}{P} \quad (44) \]

\[ d_A = \frac{\int_0^{R_2} 2\pi r w(r) \bigg|_{p=0} dr}{V} \quad (45) \]

The acoustic mass of the diaphragm is determined by the lumped kinetic energy of the oscillating diaphragm

\[ M_{aD} = \frac{2\pi \rho D}{\Delta V} \int_0^{R_e} (w(r))^2 r dr \quad (46) \]

To obtain the transverse deflection of the piezoelectric unimorph, which is a compound of the inner region inclusive of the piezoceramic layer, and the outer region which excludes the piezoceramic layer. The resulting expressions for both inner and outer region is as formulated in (38) and (39) respectively, with the matrices constants adopted in these equations calculated as in Table 3.1 and equations (47) to (60)

\[ w_0^{(1)}(r) = b_1^{(1)} \left( \frac{r^2 - R_1^2}{2} \right) - \frac{P \bigg| \begin{array}{c} r^4 - R_1^4 \end{array} \bigg|}{64 D_{11}^{(1)}} + b_1^{(2)} \left[ \frac{R_1^2 - R_2^2}{2} - R_2^2 \ln \frac{R_1}{R_2} \right] + \frac{P}{64 D_{11}^{(2)}} \left[ 4R_2^4 \ln \frac{R_1}{R_2} - R_1^4 + R_2^4 \right] \quad (47) \]
\[ w^{(2)}_0(r) = b_1^{(2)} \left[ \frac{r^2 - R_2^2}{2} - R_2^2 \ln \frac{r}{R_2} \right] + \frac{P}{64D_{11}^{(2)}} \left[ 4R_2^4 \ln \frac{r}{R_2} - r^4 + R_2^4 \right] \] (48)

**Table 3.1 Matrices [Q], [A], [B] and [D]**

<table>
<thead>
<tr>
<th>Matrix Type</th>
<th>Matrix Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stiffness Matrix</strong></td>
<td>[ [Q] = \frac{E}{1 - \nu^2} \begin{pmatrix} 1 &amp; \nu \ \nu &amp; 1 \end{pmatrix} ]</td>
</tr>
<tr>
<td><strong>Extension Stiffness Matrix</strong></td>
<td>[ [A] = \int_{z_1}^{z_2} [Q] , dz ]</td>
</tr>
<tr>
<td><strong>Coupling Matrix</strong></td>
<td>[ [B] = \int_{z_1}^{z_2} [Q]z , dz ]</td>
</tr>
<tr>
<td><strong>Flexural Stiffness Matrix</strong></td>
<td>[ [D] = \int_{z_1}^{z_2} [Q]z^2 , dz ]</td>
</tr>
</tbody>
</table>

\[ A_{11}^{(1)} = \int_{z_1}^{z_2} \frac{E_1}{1 - \nu_1^2} \, dz \] (49)

\[ A_{12}^{(1)} = \int_{z_1}^{z_2} \frac{E_1 \nu_1}{1 - \nu_1^2} \, dz \] (50)

\[ A_{11}^{(2)} = \int_{z_1}^{z_2} \frac{E_2}{1 - \nu_2^2} \, dz \] (51)

\[ B_{11}^{(1)} = \int_{z_1}^{z_2} \frac{E_1}{1 - \nu_1^2} z \, dz \] (52)

\[ B_{12}^{(1)} = \int_{z_1}^{z_2} \frac{E_1 \nu_1}{1 - \nu_1^2} z \, dz \] (53)
The acoustic resistance of the piezoelectric diaphragm, representative of the losses attributed to the damping effects present, is given as

$$R_{aD} = 2\zeta \sqrt{\frac{M_{aD}}{C_{aD}}}$$  (61)

where $\zeta$ is the diaphragm’s damping coefficient, attainable experimentally in Appendix D: Damping Coefficient of Piezoelectric Diaphragm. The MATLAB code for the diaphragm model can be found in Appendix A: MATLAB Code for Lumped Element Model.
3.2 Multiple-Diaphragm Actuator Model

While there may be a limitation to the maximum achievable velocity output for any single-orifice single-diaphragm SJA with an AC voltage input, inclusion of additional diaphragms into the SJA can serve to boost the velocity output given a similar cavity and orifice layout. High velocity output is desirable as it was found that a jet-to-free stream velocity ratio of at least one was necessary for the SJs to have sufficient momentum to penetrate the boundary layer and influence the potential flow above it for cases of fully-attached flows\(^\text{[13]}\). The multiple-diaphragm SJA equivalent circuit can be constructed by adding more diaphragm impedance in series to the single-diaphragm circuit as shown in Figure 3.3.

![Figure 3.3 Equivalent Circuit Representation of a Single-Orifice Dual-Diaphragm Piezoelectrically-driven SJA](image)

In retrospect, the acoustic impedance of the diaphragm is estimated with the electroacoustic model and in order to cater to the case whereby more than one diaphragms are used, the fundamental equation has to be modified accordingly in (62) for \(n\) number of diaphragms

\[
\frac{Q_{\text{out}}(s)}{\sum_{i=1}^{n} V_{\text{ac},i}(s)} = \frac{\sum_{i=1}^{n} \phi_{i} C_{aD,i} s}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + 1}
\]  

(62)
For cases where all diaphragms are similar, the acoustic impedances in series can be lumped into a single overall diaphragm acoustic impedance term,

\[ Z_{aD,T} = R_{aD,T} + sM_{aD,T} + \frac{1}{sC_{aD,T}} \]  

(63)

where \( R_{aD,T}, M_{aD,T}, \) and \( C_{aD,T} \) are expressed in (64) to (66),

\[ R_{aD,T} = \sum_{i=1}^{n} R_{aD,i} \]  

(64)

\[ M_{aD,T} = \sum_{i=1}^{n} M_{aD,i} \]  

(65)

\[ R_{aD,T} = \left( \sum_{i=1}^{n} \frac{1}{C_{aD,i}} \right)^{-1} \]  

(66)

Thus, for a multiple-diaphragm SJA with \( n \) number of similar diaphragms operating in parallel circuit, the frequency response function with similar diaphragms operating in phase with similar \( V_{pp} \) voltage input condition, can be simplified as

\[ \frac{Q_{out}(s)}{V_{ac}(s)} = \frac{n \phi C_{aD,T}s}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + 1} \]  

(67)

where

\[ R_{aD,T} = 2R_{aD} \]  

(68)

\[ M_{aD,T} = 2M_{aD} \]  

(69)

\[ C_{aD,T} = \frac{C_{aD}}{2} \]  

(70)
3.3 Multiple-Orifice Actuator Model

By increasing the number of orifices used in the SJA, and each orifice sufficiently spaced apart, the total circulation produced can be augmented\(^{[37]}\), as an increase in the number of orifices corresponds to a decrease in the orifice diameter and thus capable of improving the efficiency of the SJA. Accordingly, given similar input conditions, a multiple-orifice SJA in the form of a distributed Helmholtz resonator can generate a much high momentum and circulation compared to a single-orifice configuration.

The equivalent circuit for dual independent-orifices SJAs can be adjusted by attaching parallel orifice impedance branch, as shown in Figure 3.4 and 3.5 for single-diaphragm and dual-diaphragm configurations respectively.

---

**Figure 3.4 Single-Diaphragm Dual-Orifice SJA Equivalent Circuit Representation**

**Figure 3.5 Dual-Diaphragm Dual-Orifice SJA Equivalent Circuit Representation**
From the LEM perspective, in the absence of vortex interaction between orifices under quiescent condition, the net acoustic impedance of the orifices is effectively reduced by introducing more orifices. The effective orifice acoustic impedance for $n$ number of orifices can be expressed as

$$Z_{O,eff} = \left( \sum_{i=1}^{n} \frac{1}{Z_{O,i}} \right)^{-1}$$  \hspace{1cm} (71)

Hence, for dual-orifice SJAs, the effective orifice acoustic impedance can be written as

$$Z_{O,eff} = \frac{Z_{O,1}Z_{O,2}}{Z_{O,1} + Z_{O,2}}$$  \hspace{1cm} (72)

where the subscript 1 and 2 denotes the first and second orifice respectively.

For $n$ similar orifices, the effective acoustic impedance is simply a scale-down of a single orifice acoustic impedance by magnitude $n$

$$Z_{O,eff}(s) = \frac{(R_{aN} + R_{aO}) + s(M_{aN} + M_{aRad})}{n}$$  \hspace{1cm} (73)

and this proves to be a reasonable estimate for a thin perforated plate, as detailed by Beranek\cite{38}.

The updated orifice impedance is used in frequency response equation for a multiple-orifice single-diaphragm and dual-diaphragm SJA in (74) and (75) respectively,

$$\frac{Q_{out}(s)}{V_{ac}(s)} = \frac{\phi C_{aD}s}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + 1}$$  \hspace{1cm} (74)

$$\frac{Q_{out}(s)}{V_{ac}(s)} = \frac{2\phi C_{aD,7}s}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + 1}$$  \hspace{1cm} (75)

Equipped with all the required lumped element parameters, the fundamental frequency response equation is then solved iteratively by refining the orifice nonlinear acoustic resistance term. See Appendix A: MATLAB Code for Lumped Element Model.
However, in the event that the orifices are too close to each other and per se, allowing the orifices to start interacting with each other, the amount of air mass outside the orifice region that can be perturbed by the oscillatory motion of air within the orifice will be reduced ultimately. Hence for an orifice plate with high porosity, the inertial end correction will be lower and hence the corresponding orifice mass reactance will reduce. Considering the interactions between two adjacent orifices, the inertial end correction\textsuperscript{[39]} can be modified as

\[
h_c = 2 \left( \frac{4D_\Omega}{3\pi} \right) \left( 1 - \sqrt{\frac{\sigma_o}{2}} \right) \tag{76}
\]

where \(\sigma_o\) is the porosity of the perforate, defined as ratio of open area of the perforate, also known as the area of the orifice, to the total encompassed area, as shaded in blue in Figure 3.6.

![Figure 3.6 Perforation of an Orifice Plate](image)

### 3.4 Flow Profile Equations

Through the fundamental assumptions that the fluid of concern is incompressible, the flow is considered laminar if the acoustic Reynold’s number or perforate constant \(x_a\) is of a low value\textsuperscript{[40]}.

This parameter \(x_a\) is a strong determining factor for the way whereby acoustic wave is propagated, and for a circular orifice it is given as

\[
x_a = r \sqrt{\frac{\omega \rho_0}{\mu}} \tag{77}
\]
and similarly for the slot orifice,

$$ x_a = \frac{W_0}{2} \sqrt{\frac{\omega \rho_0}{\mu}} $$

(78)

As long as the condition for acoustic Reynold’s number $x_a < 2$ is satisfied, where the flow condition falls within the regime of Poiseuille flow, the flow is dominated by viscous effects. For flows with high perforate constant, where $x_a \approx 10$ and above, the flow is dominated by inertia effects instead, which is known as Helmholtz conditions. As long as the condition for a Poiseuille flow is fulfilled, the orifice neck acoustic impedances derived in the earlier sections can be applied for multiple-orifice configurations. For Poiseuille flow, the flow velocity profile at the orifice exit is parabolic while for high $x_a$, the velocity profile is characterized by high peaks near the edge and low, flat profile at the center.

By adopting the assumption for a fully-developed flow, the centerline velocity is related to the volume flow rate by

$$ u_O = \frac{Q_{out}}{n(S_n)} $$

(79)

for straight orifices, whereby $n$ is the total number of similar orifices perforated on the SJA. Equation (79) holds true even for single-orifice SJA$s$ where $n = 1$, as long as the flow is fully-developed at sufficiently low driving frequency.

At higher frequencies, the orifice flow velocity profile is modeled as a circular duct or channel driven by an oscillatory pressure gradient. As given by White[41], the flow profile equation for a circular orifice is

$$ u(r, t) = j \frac{\Delta P_{out}}{\omega \rho_0 h} \left\{ 1 - \frac{J_0 \left( \sqrt{-j \frac{\omega r^2}{\nu}} \right)}{J_0 (St \sqrt{-j})} \right\} e^{j\omega t} $$

(80)
where $J_0$ is a Bessel function of zero order, and $St$ is the Stokes number defined as

$$St = \sqrt{\frac{\omega a_0^2}{\nu}}$$

(81)

and Poiseuille flow occurs when the Stokes number approaches zero.

Similarly, the flow profile equation for a slot is

$$u(x, t) = \frac{\Delta P_{out}}{\omega \rho_0 h} \left\{ 1 - \frac{\cosh \left( x \sqrt{\frac{j}{\nu}} \right)}{\cosh \left( St \sqrt{j} \right)} \right\} e^{j\omega t}$$

(82)

where $x$ indicates the length-wise position along the slot from the center axis.
CHAPTER 4: EXPERIMENTAL VALIDATION

4.1 Design of Actuator Test Rig

In order to validate the LEM model, a configurable SJA was designed and fabricated to perform a parametric study of the correlation between its geometries and jet flow characteristics. On top of incorporating conventional single orifice and single diaphragm SJA configurations to the test rig, multiple orifices and dual diaphragms had also been added in order to extend the validation of LEM to encompass more novel designs.

The SJA test rig can be viewed separately in three basic components for design consideration, namely the orifice plate, cavity volume, and the diaphragm plate. For the orifice plate, Figure 4.1 illustrates a few of the orifice plate variations adopted for the test rig. Clockwise from top left, are orifice plates for single slot, single circular orifice, double slots, and double circular orifices.

![Figure 4.1 Illustrations of Orifice Plates used for Test Rig](image)

It is possible to increase the efficiency of the SJA by more than 300% through use of multiple orifices, provided that there is sufficient spacing between the orifices and the orifice geometry induces optimum dimensionless stroke length. The separation distance between the orifices located on the orifice plate is of paramount importance as the interaction between vortices developed by each orifice may have an undesirable effect of partially canceling out vortex ring circulation of each orifice, due to opposing sign of vorticity from interacting adjacent rings. This will result in less total circulation if there is inadequate spacing between the orifices.
Figure 4.2 Spacing Between Adjacent Orifices

As specified by Riazi\textsuperscript{[37]}, there are three different outcomes for orifice spacing, namely, (i) immediate interaction of adjacent vortex rings in close proximity to the orifices exit, (ii) delayed interaction of adjacent vortex rings as they curve towards each other further downstream away from the orifices, and (iii) no interaction between the adjacent vortex rings as they propagate in a straight path away from the orifices. The spacing between the orifices excluding the orifice itself, as shown in Figure 4.2 where this spacing is defined as \((L_{\text{sep}} - D_o)\) or \((L_{\text{sep}} - W_o)\), of \(1.5 \cdot L \cdot D_o\) as given in the aforementioned third outcome, is probably adequate for the prevention of interaction between the vortex rings, as determined by Riazi\textsuperscript{[37]}. The dimensionless stroke length \(L\) is defined as

\[
L = \frac{L_s}{h_o}
\]  

(83)

where \(L_s\) is the stroke length given by

\[
L_s = \frac{U_{\text{max}}}{\pi f}
\]  

(84)

and \(h_o\) is the characteristic length of the orifice, which is equivalent to orifice diameter for circular orifice, and slot width for slot orifice. Hitherto, a minimum spacing of \(L \cdot D_o\) is desired for the vortex structures to formed properly and independently.
If the orifices can be considered independently, whereby the inter-orifice spacing is sufficiently large, i.e. orifice opening cross-sectional area is considerably smaller than the cavity cross-sectional area, the resonance frequency of the multiple-orifice SJA can be estimated based on the simplified SJA model as elaborated in the earlier chapter.

For multiple-orifice SJA, the maximum dimension of the orifice has to be kept within the limit prescribed by a low perforate constant in order for the flow regime to be considered as a Poiseuille flow. For a multiple-orifice SJA with circular orifices of diameter 2mm, the acoustic Reynold’s number is estimated to be 0.0074. Similarly, the acoustic Reynold’s number for a multiple-orifice SJA with 15mm length slots is 0.055. The acoustic Reynold’s number for both circular orifice and slot is very low, thus it is reasonable to assume that the orifice flow falls under the Poiseuille flow regime.

According to Craggs and Hildebrandt[42], for a linear acoustic domain with independent perforations, by using Finite Element Method (FEM) to solve the Navier-Stokes equation, one-dimensional acoustic wave propagation can be determined for orifice or tubes of varying shapes. Considering solely on the axial velocity component of the orifice flow, the Newton’s equation for a circular or slot orifice can be expressed in the form

\[-\frac{\partial p}{\partial z} = j\omega \rho_e \bar{U} + \sigma_e \bar{U}\]  \hspace{1cm} (85)

where \(z\) denotes the specific impedance \(z = p/\nu\), \(\rho_e\) is the real effective density, and \(\sigma_e\) is the real effective flow resistivity. Both the density ratio \(\rho_e/\rho_0\) and resistivity ratio \(\sigma_e \bar{r}^2/\mu\) are shown to be functions of \(x\), where \(\bar{r}\) is the hydraulic radius of the orifice, defined as twice the orifice cross-sectional area divided by its perimeter. Due to the fact that the acoustic Reynold’s number for the orifices implemented in the test rig is very small, i.e. \(x_a \approx 0\), the density ratio for a circular orifice and slot is given as 1.333 and 1.2 respectively, and the extreme resistivity ratio for small \(x_a\) value is 12. For large \(x_a\), the density ratio is reduced to a constant value of 1.15 for all cross-sectional shapes, and the resistivity ratio is proportionate to \(x_a\)[43].

43
Additional considerations relevant to the design of the test SJA to suit implementation on an UAV wing were made. Firstly, the SJA cavity has to be sufficiently thin to ease installation without causing overly larger orifice depth due to the curvature of the airfoil. As illustrated in Figure 4.3, the slim SJA on the left fits the aircraft wing with minimal interface gap between the upper wing surface and the upper cavity region, while the wider cavity SJA on the right causes a much noticeable gap between the wing surface and the cavity entrance. Secondly, the number of diaphragms used must be sufficient to produce high velocity flow, yet must not be excessive due to limitations to the power requirement for an array of SJAs. Keeping in mind these conditions the test SJA will be made vertically thin (horizontally compact) with a maximum of two diaphragms in operation.

![Figure 4.3 SJAs of Different Cavity Compactness Mounted in Wing](image)

Structurally, the SJA test rig used in the test is composed of a cavity plate which includes the orifice, and sandwiched by two diaphragm plate for clamping the PZT disc on each side of the cavity. Figure 4.4 illustrates the cross-sectional view of the SJA with dual orifices. Detailed CAD drawings can be found in Appendix B: CAD Drawings of Configurable Synthetic Jet Actuator.

Piezoelectric discs of 27mm overall diameter were used in the test, and the corresponding technical specifications are found in Table 4.1. These discs were clamped 1mm radially inwards along their circumferences by two aluminum plates, which were in turn bolted onto the main cavity plate containing the orifice plate at each corner of the rectangular clamp plate, through the use of M5 bolts to ensure the plates were clamped tightly. Hence,
the effective diameter of the diaphragm, $D_{\text{eff}}$, is 25mm. An illustration of the SJA is as shown in Figure 4.5.

Figure 4.4 Cross-Sectional Schematic of Configurable SJA

Figure 4.5 3D Render of SJA Used in Experiment
To validate the LEM model, the configurable SJA was designed to vary in terms of its orifices characteristics and cavity volume. Table 4.2 shows all the configurations tested for the SJA test rig, with the geometries and dimensions specified for each cases. The frequency response curve for these cases were predicted using LEM to ensure that the first resonance peak is less than 2kHz, to account for the limitation of the LEM model as it is dependent on the lower frequency range in order for the underlying assumptions to hold true. Given that the speed of sound is 343m/s for an environment with temperature of 20°C at sea level condition, the minimum acoustic wavelength is 171.5mm for a upper frequency bound of 2kHz, is much greater than the largest dimension of the SJA, which is the cavity’s interior cross-sectional length of 30.5mm. Higher frequencies are undesirable in this model as the peak velocity is estimated from Poiseuille flow, and this relationship is only valid at low frequencies for ducts with fully-developed flow. Hence the error will be amplified for higher frequencies.

<table>
<thead>
<tr>
<th>Table 4.1 Piezoelectric Disc Technical Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass Shim Diameter</td>
</tr>
<tr>
<td>Brass Shim Thickness</td>
</tr>
<tr>
<td>Piezoceramic Diameter</td>
</tr>
<tr>
<td>Piezoceramic Thickness</td>
</tr>
<tr>
<td>Electrode Diameter</td>
</tr>
<tr>
<td>Resonant Frequency</td>
</tr>
<tr>
<td>Resonant Impedance</td>
</tr>
<tr>
<td>Capacitance</td>
</tr>
<tr>
<td>Elastic Modulus of Brass</td>
</tr>
<tr>
<td>Poisson Ratio of Brass</td>
</tr>
<tr>
<td>Density of Brass</td>
</tr>
<tr>
<td>Elastic Modulus of Piezoceramic</td>
</tr>
<tr>
<td>Poisson Ratio of Piezoceramic</td>
</tr>
<tr>
<td>Density of Piezoceramic</td>
</tr>
<tr>
<td>D31 Piezoelectric Constant</td>
</tr>
</tbody>
</table>

The first resonance peak corresponds closely to the Helmholtz frequency, while the second distinctive peak, which is greater than 4kHz, matches the diaphragm’s resonance fundamental frequency which would not be covered in the report.
The Helmholtz frequency can be estimated by

\[ f_H = \frac{1}{2\pi} \sqrt{\frac{1}{C_{ac}(M_{aN} + M_{aRad})}} \]  \hspace{1cm} (86)

while the short-circuit piezoelectric diaphragm’s natural frequency is

\[ f_D = \frac{1}{2\pi} \sqrt{\frac{1}{C_{ad}M_{aD}}} \]  \hspace{1cm} (87)

Also, through voltage input of at least 100 Vpp, higher peak velocities are achievable in order to enable a more accurate measurement of the velocity value.

While only certain cases are highlighted in this report, results for all of the cases prescribed in Table 4.1 can be found in Appendix E: Experimental and LEM Results for Various Configurations.
Table 4.2 SJA Test Configurations

<table>
<thead>
<tr>
<th>Case</th>
<th>Orifice Type</th>
<th>Orifice Depth</th>
<th>Cavity Height</th>
<th>Cavity Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Single Circular Orifice, φ1 mm</td>
<td>2 mm</td>
<td>5 mm</td>
<td>3.965.9 mm³</td>
</tr>
<tr>
<td>2</td>
<td>Single Circular Orifice, φ1 mm</td>
<td>2 mm</td>
<td>8 mm</td>
<td>6.050.9 mm³</td>
</tr>
<tr>
<td>3</td>
<td>Single Circular Orifice, φ2 mm</td>
<td>2 mm</td>
<td>5 mm</td>
<td>3.965.9 mm³</td>
</tr>
<tr>
<td>4</td>
<td>Single Circular Orifice, φ2 mm</td>
<td>2 mm</td>
<td>8 mm</td>
<td>6.050.9 mm³</td>
</tr>
<tr>
<td>5</td>
<td>Single Slot Orifice, length 5 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>5 mm</td>
<td>3.965.9 mm³</td>
</tr>
<tr>
<td>6</td>
<td>Single Slot Orifice, length 5 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>8 mm</td>
<td>6.050.9 mm³</td>
</tr>
<tr>
<td>7</td>
<td>Single Slot Orifice, length 10 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>5 mm</td>
<td>3.965.9 mm³</td>
</tr>
<tr>
<td>8</td>
<td>Single Slot Orifice, length 10 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>8 mm</td>
<td>6.050.9 mm³</td>
</tr>
<tr>
<td>9</td>
<td>Single Slot Orifice, length 15 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>5 mm</td>
<td>3.965.9 mm³</td>
</tr>
<tr>
<td>10</td>
<td>Single Slot Orifice, length 15 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>8 mm</td>
<td>6.050.9 mm³</td>
</tr>
<tr>
<td>11</td>
<td>Double Circular Orifices, φ1 mm</td>
<td>2 mm</td>
<td>5 mm</td>
<td>3.965.9 mm³</td>
</tr>
<tr>
<td>12</td>
<td>Double Circular Orifices, φ1 mm</td>
<td>2 mm</td>
<td>8 mm</td>
<td>6.050.9 mm³</td>
</tr>
<tr>
<td>13</td>
<td>Double Slots, length 5 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>5 mm</td>
<td>3.965.9 mm³</td>
</tr>
<tr>
<td>14</td>
<td>Double Slots, length 5 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>8 mm</td>
<td>6.050.9 mm³</td>
</tr>
<tr>
<td>15</td>
<td>Triple Circular Orifices, φ1 mm</td>
<td>2 mm</td>
<td>10 mm</td>
<td>7.441.0 mm³</td>
</tr>
<tr>
<td>16</td>
<td>Triple Slots, length 10 mm, width 0.5 mm</td>
<td>2 mm</td>
<td>10 mm</td>
<td>7.441.0 mm³</td>
</tr>
</tbody>
</table>

4.2 Experimental Set-Up

The SJA was set up as shown in Figure 4.6 and 4.7, with the Dantec Dynamics Probe Type 55P16 hot-wire probe positioned just slightly above the orifice opening. The probe was attached to the Dantec Dynamics Miniature CTA 54T30, with the probe temperature maintained at 214°C for the overheat setting. The CTA was then outputted to the National Instruments NI USB-6008 data acquisition.

Wavetek 20MHz Synthesized Function Generator Model 90 was used to create a sinusoidal AC voltage input and amplified to the SJA using two PiezoDrive PDX Voltage
Amplifier as amplifier and inverting amplifier, resulting in a 20 times amplification of the voltage signal to the piezoelectric diaphragms. The two piezoelectric diaphragms were connected in parallel to the voltage source, i.e. directly connected to the amplifiers, if dual-diaphragm operation is desired for the test.

Positioned directly above the orifice, the peak centerline velocity of the synthetic jet was measured and recorded, with AC input frequency varied from 100Hz to 2kHz to capture the frequency response of the test SJA, with input voltage of at least 100 Vpp. Figure 4.8 shows the placement of the hot-wire probe above the SJA orifice.

Figure 4.6 Schematic of Experimental Set-up
Figure 4.7 Actual Experimental Set-up

Figure 4.8 Placement of Hot-wire Probe
4.3 Constant Temperature Hot-Wire Anemometry

In Constant Temperature Anemometry (CTA), also known as Thermal Anemometry, the fluid flow is measured using a hot-wire probe. CTA is very suited for measurement of very high fluctuations at a point, or high turbulence, due to its inherently high temporal resolution especially for measuring small flow eddies.

The principles behind CTA measurement is based on the application of cooling effect on a heated cylindrical object, i.e. the probe wire, via fluid flow. Figure 4.9 shows the process of CTA measurement. By ensuring that wire resistance is kept constant, a servo amplifier within the CTA’s Wheatstone bridge circuit balances by adjusting the current to the sensor or probe to maintain the probe temperature.

![Figure 4.9 Constant Temperature Anemometry Measurement][44]

Since the convective heat transfer, $Q_h$, from the probe wire is a function of the flow velocity of interest, $U$, a basic relationship between $Q_h$ and $U$ is as formulated using King’s Law\[45\]

$$Q_h = (T_W - T_0)A_W h = A_T + B_T U^n$$  (88)

where $(T_W - T_0)$ is the wire over-temperature, $A_W$ is the exposed surface area of the probe’s wire, $h$ is the heat transfer coefficient, $A_T$ and $B_T$ are calibration constants, and $n \approx 0.5$. 

[44]: Image of Constant Temperature Anemometry Measurement
[45]: King’s Law
As the CTA’s Wheatstone bridge voltage is dependent on the fluid velocity and temperature, a 1K change will result in an error of approximately 2% in velocity measurement. However, the difference in temperature throughout the duration of the experiment can be taken as minimal, hence the error margin is relatively small. The temperature correction equation can be found in *Appendix C: Hot-Wire Anemometry*.

The relationship between the measured voltage and flow velocity can be described as an exponential function (see *Velocity Calibration – Exponential Function in Appendix C: Hot-Wire Anemometry*) or linearized as a 4\textsuperscript{th} order polynomial as seen in Figure 4.10.

![Figure 4.10 Flow Velocity Calibration](image_url)
CHAPTER 5: ANALYSIS AND DISCUSSION

5.1 Conventional Actuator Experimental and LEM Results

Based on the LEM model by Gallas\textsuperscript{[24]}, the LEM predicted data is supported by experimental results in Figure 5.1 and 5.2 for Case 2 and 7 respectively, for input voltage of 100 Vpp and 200 Vpp, with up to two diaphragms in operation. The LEM model is capable of reflecting the frequency response of the SJA by predicting the resonance peak, and to certain extent, the amplitude trend for varying voltage input and diaphragms in operation.

![Single 1mm Diameter Orifice, 8mm Cavity Height](image)

**Figure 5.1 Case 2 Experimental and LEM Results**

For the case of dual-diaphragm, whereby two diaphragms were activated during the test, the frequency trend is similar to that of a single-diaphragm configuration, except with a much higher peak centerline velocity. In the LEM results generated for Case 2 and 7, the
peak centerline velocities for single diaphragm inoperative with 200 Vpp A.C. voltage input are similar to those of two diaphragms in operation with 100 Vpp input.

**Figure 5.2 Case 7 Experimental and LEM Results**

Despite reasonably predicting the frequency trend for suitably small orifice dimensions like Case 2 and 7, the estimation of the orifice acoustic radiation mass for a long slot such as Case 9 and 10 seems to be inexact as the slot size is actually considerably large for the orifice plate, thus the formulation of the slot acoustic radiation mass term will not be accurate. Figure 5.3 shows the results for Case 9 where there is an underestimation of the Helmholtz resonance peak for both LEM prediction and the experimental data, which can be attributed to an inaccuracy in the acoustic nonlinear reactance term. Hence, for such cases, it is necessary to adopt an alternate inertial end correction to better estimate the longer slot’s acoustic radiation mass.
5.2 Diaphragm Performance

In the LEM model, it is assumed that the piezoelectric diaphragms used in the dual-diaphragm operations are similar, in terms of its material properties and related performance. However, such ideal case is non-existent in reality. It has been determined that the SJA performance based on various diaphragms of the same type can differ more than 5% at resonance. Figure 5.4 compares the single diaphragm inoperative SJA with two different piezoelectric discs of the same type driving one of the SJA test configuration. Diaphragm 1 and Diaphragm 2 denotes the two separate piezoelectric diaphragms utilized and operated individually in a single-diaphragm SJA configuration, both of which are of similar geometric and material properties.

By operating the SJA with different pieces of the piezoelectric discs, despite their similarities in geometries and material, slight discrepancies can be observed. Due to this dissimilar performance by different diaphragms of the same type, experimentally, the peak
velocities produced with one diaphragm in-operative driven at 200 Vpp and that of two-diaphragms in operation at 100 Vpp shows a difference in velocity values, unlike the LEM-predicted trend where both cases will attain similar velocities for a given frequency.

![Figure 5.4 Comparison of Experimental Results Based on Single-Diaphragm SJA with Different Diaphragms](image)

5.3 Comparison of Experimental and LEM Results

By introducing multiple orifices to the orifice plate, based on the simplified multiple-orifice LEM model elaborated in the earlier section, the overall orifice acoustic impedance is lowered by a factor of \( n \), whereby \( n \) is the total number of similar orifices on the orifice plate. A reduced orifice acoustic impedance will enhance the total flow velocity of the SJA in the absence of mutual interactions between adjacent orifices, which in turn has the potential to boosts the total momentum imparted to the external flow.
The simplified multiple-orifice LEM model is capable of projecting the frequency response trend of the SJA, especially the first resonance peak related to the Helmholtz resonance, as shown in Figure 5.5 and 5.6 for Case 12 and 14 respectively. However, in certain instances like Case 14, the peak velocity estimation vastly falls short of the actual experimental values for AC input voltage of 200Vpp. The underestimation in peak velocities may be due to the fact that other real effects are not taken into consideration in this simplified model. In order to properly account for these additional effects, the impedance terms have to be relooked at and corrected accordingly. Good estimation of peak centerline velocity for results based on 100Vpp input demonstrated that quintessentially, the discrepancy seen in 200Vpp is attributable to the amplification effect by $V_{ac}$ in the LEM general equation presented early.

![Double 1mm Orifice, 8mm Cavity Height](image)

**Figure 5.5 Case 12 Experimental and LEM Results**

Despite the inaccuracy for large AC input voltage when it comes to amplitude for high input voltage cases, the simplified model still serves reasonably well for a low-order model to predict the resonance frequency especially in the initial stage of SJA design.
5.4 Design Considerations

The designing of SJA can be decomposed simply into several components: (i) cavity volume, (ii) orifice depth, (iii) characteristic dimension of orifice (diameter for circular orifice, and slot length for slot orifice), (iv) number of diaphragms, and (v) number of orifices. Each of these components can be varied to suit the design frequency limit and optimized for maximum achievable momentum impartation.

The resulting shift in the Helmholtz resonance peak attained by varying the geometry of the cavity and the orifice, which can be predicted from equation (86) mentioned in Chapter Four, where the acoustic resonance frequency is directly dependent on the cavity acoustic compliance, the linear acoustic mass and the acoustic radiation mass of the orifice,

$$f_H = \frac{1}{2\pi} \sqrt{\frac{1}{C_{ac}(M_{aN} + M_{aRad})}}$$
Hence, a change in the cavity volume will affect its acoustic compliance and subsequently the Helmholtz frequency. Likewise, a change in orifice depth will affect its linear acoustic mass and a change in orifice shape and size will affect both its linear acoustic mass and acoustic radiation mass, and both orifice impedances will also influence the Helmholtz resonance frequency.

![Graph showing the effect of different cavity volumes on peak centerline velocity and frequency.](image)

*Figure 5.7 Comparison of Effect of Different Cavity Volumes*

<table>
<thead>
<tr>
<th>Orifice</th>
<th>Orifice Shape</th>
<th>Circular Orifice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Orifice</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Orifice Diameter</td>
<td>1 mm</td>
</tr>
<tr>
<td></td>
<td>Orifice Depth</td>
<td>2 mm</td>
</tr>
<tr>
<td>Diaphragm</td>
<td>Number of Diaphragm in Operation</td>
<td>1</td>
</tr>
<tr>
<td>Input Condition</td>
<td>AC Voltage</td>
<td>100 Vpp</td>
</tr>
</tbody>
</table>

Table 5.1 Fixed Geometric Properties of SJAs Simulated in Figure 5.7

For an acoustic spring such as the SJA cavity, the acoustic compliance is directly proportional to the acoustic volume of interest. Hence, based on the LEM model, it is intuitive that by having a reduced cavity volume, the velocity output can be increased, albeit a shift in the resonance peak to that of a higher frequency as shown in Figure 5.7,
with its simulated SJA geometries tabulated in Table 5.1, predictable from the Helmholtz frequency equation mentioned earlier.

The selection of an appropriate orifice is essential in optimizing the SJA design. The choice of having a slot orifice or a circular orifice is dependent on the requirements specified, as circular orifice provides a much higher efficiency while the slot orifice enhances the total momentum imparted to the external flow due to higher air mass exit rate from a relatively larger slot area exposed to the freestream. A combination of orifice depth and size also affects the velocity output of the SJA, due to the fact that proper balance has to be realized between orifice depth and size in order to attain suitable dimensions producing optimal velocity flow at the resonance peak. In the conventional single-orifice single-diaphragm configuration, as shown in Figure 5.8 for a typical circular orifice, with its corresponding SJA geometries as detailed in Table 5.2. The increase in orifice depth will indisputably decrease the peak velocity at the same rate, thus it can be observed that the amalgamation of the SJ is easier to be achieved for a device with a smaller orifice depth.

In the case of a circular orifice or a slot, the increase in orifice diameter or length will result in a shift of the resonance peak to that of a higher frequency, tied with a slight increase in peak velocity, as validated experimentally. Figure 5.9 and 5.10 highlighted the resonance shift and variations in peak velocity for an increase in circular orifice diameter and slot length respectively. The corresponding geometries of the SJAs for Figure 5.9 and 5.10 are detailed in Table 5.3 and 5.4 respectively.

Bearing in mind the design considerations for orifice geometry and cavity volume, in the early stage of the project, with the intention of designing a typical single-orifice single-diaphragm SJA for implementation on a small UAV for a fixed 10mm length slot orifice of 2mm depth, cavity height of 8mm with a total cavity volume of 6050.9mm$^3$ is chosen in order to ensure that the first resonance frequency or the Helmholtz frequency, to not be exceedingly high for a low Strouhal number orifice flow, and yet producing sufficiently high flow velocity at the orifice exit. As a rule of thumb, designing of small SJAs operating at high frequencies exceeding 1kHz should be avoided, due to the presence of stronger
nonlinearities related to large pressure oscillations within and around the orifice, in addition to higher compressibility effects. Slot orifice is adopted with the specific purpose of introducing higher rate of imparting momentum to the flow when an array of SJAs is in operation.

![Figure 5.8 Comparison of Effect of Different Orifice Depths](image)

**Table 5.2 Fixed Geometric Properties of SJAs Simulated in Figure 5.8**

<table>
<thead>
<tr>
<th>Orifice</th>
<th>Orifice Shape</th>
<th>Circular Orifice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Orifice</td>
<td>1</td>
</tr>
<tr>
<td>Cavity</td>
<td>Orifice Diameter</td>
<td>1 mm</td>
</tr>
<tr>
<td></td>
<td>Cavity Volume</td>
<td>3,965.9 mm³</td>
</tr>
<tr>
<td>Diaphragm</td>
<td>Number of Diaphragm in Operation</td>
<td>1</td>
</tr>
<tr>
<td>Input Condition</td>
<td>AC Voltage</td>
<td>100 Vpp</td>
</tr>
</tbody>
</table>

The corresponding experimental results conducted under quiescent condition is as shown in Figure 5.11 with input AC voltage at 100Vpp and 200Vpp. The maximum peak centerline velocity for such configuration was close to 9m/s at around 950Hz, with a voltage input of 200Vpp. The Reynolds number and Strouhal number in this configuration can be calculated to be 5700 and 1 respectively. The Reynolds number should generally
be larger than 50 for low viscous losses while the Strouhal number should not be more than 1 for less unsteadiness and more directivity in the SJ produced\cite{14}. To increase the peak centerline velocity further, higher AC voltage input has to be given, which is definitely not a lucrative option for a small UAV, where high voltage source requirement might impose an operational issue.

Figure 5.9 Comparison of Effect of Single Circular Orifice with Different Diameters

Table 5.3 Fixed Geometric Properties of SJAs Simulated in Figure 5.9

<table>
<thead>
<tr>
<th>Orifice</th>
<th>Orifice Shape</th>
<th>Circular Orifice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Orifice</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Orifice Depth</td>
<td>2 mm</td>
</tr>
<tr>
<td>Cavity</td>
<td>Cavity Volume</td>
<td>3,965.9 mm³</td>
</tr>
<tr>
<td>Diaphragm</td>
<td>Number of Diaphragm in Operation</td>
<td>1</td>
</tr>
<tr>
<td>Input Condition</td>
<td>AC Voltage</td>
<td>100 Vpp</td>
</tr>
</tbody>
</table>

Conversely, by embracing novel design features like multiple-orifice and multiple-diaphragm, it is possible to significantly increase the maximum volumetric flow rate of the SJA. The use of independent, mutually non-interacting perforations on the orifice plate has the capacity to improve the peak centerline velocity on each of its orifices due to a
correspondingly huge drop in orifice acoustic impedances. Shown in Figure 5.12, the addition of a slot, both of which with 5mm slot length and operating at 200Vpp, results in an increase of the peak centerline velocity by approximately 1.4m/s, and this corresponds to an increase in the total volumetric flow rate by at least 140%.

![Figure 5.10 Comparison of Effect of Single Slot with Different Lengths](image)

| Table 5.4 Fixed Geometric Properties of SJAs Simulated in Figure 5.10 |
|---------------------------------|-----------------|-----------------|
| Orifice | Orifice Shape | Slot |
| Number of Orifice | 1 | |
| Orifice Depth | 2 mm | |
| Slot Width | 0.5 mm | |
| Cavity | Cavity Volume | 3,965.9 mm³ |
| Diaphragm | Number of Diaphragm in Operation | 1 |
| Input Condition | AC Voltage | 100 Vpp |
Figure 5.11 Single-Slot Single-Diaphragm SJA Experimental Result

Figure 5.12 Single-Slot and Double-Slot Single-Diaphragm SJA Experimental Results
As detailed in the earlier sections, the novel means of enhancing total flow rate of the SJA highlighted in this report is two-pronged; by increasing the number of orifices, and by increasing the number of diaphragms in operation. When considering the number of diaphragms to use, it is expedient to note that as the number of diaphragms increases, the increase in peak velocity will decrease nonlinearly, as predicted by LEM in Figure 5.13, which is based on a set of fixed geometries as detailed in Table 5.5. Also in reality, it has been shown that there is marginal difference between operating an SJA with 3 diaphragms or 5 diaphragms\cite{46} in operation. Thus for a simple hexahedral cavity with piezoelectric diaphragm prescribed at any of the hexahedron face, dual-diaphragm is the best design option as it not only can give the smallest possible cavity volume, but also augment the achievable flow velocity by almost twice of that realized by single-diaphragm SJA.

![Figure 5.13 Effect of Different Number of Diaphragms in Operation](image)

<table>
<thead>
<tr>
<th>Orifice</th>
<th>Orifice Shape</th>
<th>Circular Orifice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Orifice</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Orifice Diameter</td>
<td>2 mm</td>
<td></td>
</tr>
<tr>
<td>Orifice Depth</td>
<td>2 mm</td>
<td></td>
</tr>
<tr>
<td>Cavity Volume</td>
<td>21,952.0 mm³</td>
<td></td>
</tr>
</tbody>
</table>

Input Condition: AC Voltage 100 Vpp
For the 10mm slot length configuration examined earlier, the net effect of mounting another piezoelectric composite diaphragm to the single-orifice single-diaphragm SJA boosted the peak centerline velocity by more than 40%, as evident in Figure 5.14 when given an AC input of 200Vpp, with both similar piezoelectric composite diaphragms connected in parallel to the voltage source.

By including additional orifices and diaphragms to the initial SJA design, it is highly possible to increase the flow rate of the SJA by several folds. A dual-slot of similar length of 5mm with fixed orifice depth of 2mm, and cavity height of 8mm which corresponds to a total cavity volume of 6050.9mm$^3$, was to driven by two diaphragms in tandem, both operating without any phase lag. The use of 5mm slot, primarily, is to adjust the resonance peak to a lower frequency to counter the addition of another adjacent slot, derived from the observation that an increase in the number of orifices will also shift the resonance peak to that of a higher frequency.

![Figure 5.14 Single-Slot Single-Diaphragm and Dual-Diaphragm SJA Experimental Results](image-url)
Thus in Figure 5.15, the conventional SJA had a 10mm slot and was driven solely by a single diaphragm, while the dual-slot dual-diaphragm SJA had two 5mm slots and was driven by two similar diaphragms operating in phase, and the Helmholtz resonance peak matches closely with each other. Under an AC input of 200Vpp, the peak centerline velocity is increased by 2.3m/s, and this relates to an increase of maximum flow rate by at least 150% at resonance.

Hence, by considering the various factors affecting the overall flow velocity, a balance among these considerations can be established which caters specifically to the design requirement. The use of multiple orifices and diaphragms in the SJA design, as seen from the experimental results for dual-orifice and dual-diaphragm, represented a breakthrough in SJA design which can amplify the total flow rate achieved by a conventional SJA. These new designs show promise in UAV application especially if optimally-designed piezoelectric diaphragm can be chosen to provide very high transverse deflections with low voltage input.

![Figure 5.15 Conventional and Dual-Slot Dual-Diaphragm SJA Experimental Results](image-url)
CHAPTER 6: CONCLUSIONS AND FUTURE WORK

The LEM model discussed in this report offers a computationally economical low-order modeling of the SJA under quiescent condition, which is exceptionally convenient for the initial stage of SJA design to optimize the device geometries. Despite the inherent lack of ability to fully incorporate all real effects into the simplified model highlighted in this report, the LEM model provides a reasonably accurate prediction of the SJA frequency response of the first peak, directly attributable to the Helmholtz resonance frequency, albeit underestimation of the actual peak centerline velocity for 200Vpp voltage input.

Nevertheless the model furnished an intuitive insight into the working dynamics of the SJA by condensing the complexities into a simple electro-acoustic circuitry representation. However, the orifice acoustic radiation mass for the long slot configurations used in this project can be further refined in future in order to accurately determine the resonance peak for slots that are significantly large relative to the cavity.

Hitherto the benign effects of introducing multiple independent orifices and piezoelectric composite diaphragms were discussed and the inclusion of such novel features into the conventional designs of SJA serve to enhance flow velocity drastically. The considerations for these additional features have been expanded under the existing LEM model with decent accuracy for prediction of the first resonance peak frequency. Still, both the orifice impedances for multiple-orifice cases and the diaphragm parameters for multiple-diaphragm cases have to be refined to account more precisely for actual effects in order to estimate the peak centerline velocity with greater precision.

Additionally, as part of future work, tapered orifice instead of a straight orifice can be included into the LEM model through proper correction of the orifice impedance terms. Tapered orifice design is believed to be capable of transferring the vortex rings further downstream as they contain higher energy, in addition to higher peak velocities compared to the straight orifice adopted in this project.

Also, a separate study can be conducted to verify suitable separation distances between adjacent orifices in a perforate or multiple-orifice configuration especially for the case of
slot orifices in order to ensure that each orifice is independent and non-interacting with the other orifices. A specific guideline for spacing out multiple slot orifices will prove to be useful in future design considerations.

In conclusion, SJA based on multiple-orifice multiple-diaphragm design has the potential to enhance the device performance drastically and a more complete development on the extended LEM model in future for such design can improve its prediction to higher accuracy.
REFERENCES


APPENDIX A: MATLAB Code for Lumped Element Model

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% LUMPED ELEMENT MODEL OF SYNTHETIC JET ACTUATOR
% %
% Covered in this MATLAB code is the extended LEM model incorporating similar multiple-orifice and multiple-diaphragm. %
% Modify the SJA input parameters as required. %
% The peak centerline velocity vs frequency graph will be generated. %
% All values are in standard metric units
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear
clc

% Set this value to
% 1: Circular Orifice(s)
% 2: Slot Orifice(s)
% 3: Mixed Orifices (1 hole and 1 slot) [Measured from hole]
setConfig = 1;

% SJA Geometric Input Parameters
Do = 1e-3; % Orifice diameter (For circular orifice)
Bo = 5e-3; % Orifice length (For slot orifice)
Wo = 0.5e-3; % Orifice width (For slot orifice)
H = 10e-3; % Cavity height (amend Vc if using different design)
h = 2e-3; % Orifice depth
n = 1; % Number of orifices
nd = 1; % Number of diaphragms

% Piezoelectric Diaphragm Input Parameters
E_p = 63e9; % Elastic modulus of piezoceramic layer
v_p = 0.31; % Poisson ratio of piezoceramic layer
rho_p = 7700; % Density of piezoceramic layer
h_p = 0.25e-3; % Thickness of piezoceramic layer
R_p = 9.85e-3; % Radius of piezoceramic layer
E_s = 89.63e9; % Elastic modulus of shim
v_s = 0.324; % Poisson ratio of shim
rho_s = 8700; % Density of shim
h_s = 0.3e-3; % Thickness of shim
R_s = 12.5e-3; % Radius of shim
zeta = 0.05; % Damping coefficient of the diaphragm
d31 = -1.75e-10; % Piezoelectric constant [m/V]
Vac_val = 150; % Input ac voltage [V] (sinusoidal)

% Other Parameters
epsilon_r = 1750; % Relative dielectric constant
epsilon_0 = 8.8542e-12; % Vacuum permittivity [F/m]
Rho = 101325/(287*300); % Air density [kg/m^3]
nu = 1.6e-5; % Kinematic viscosity [m^2/s]
a_square = 1.4*287*300; % Square of sound speed [m^2/s^2]
Vc = H*(0.5*pi*(12.5e-3)^2 + (17e-3)*(25e-3) + (23e-3)*(1e-3) + 0.5*pi*(1e-3)^2) +
(1e-3)*pi*(12.5e-3)^2; % Cavity volume

cdisp = (1+nu)/2; % Cavity compliance

% Lumped Elements (Diaphragm) Definition
% C_ad - Short-circuit acoustic compliance [m^3/Pa]
% M_ad - Acoustic inductance [kg/m^4]
% R_ad - Acoustic resistance [kg/(m^4*s)]
% d_a - Effective acoustic piezoelectric coefficient [m^3/V]
% da = vol_change C_ad M_ad =
diaphragm(E_p,v_p,rho_p,R_p,h_p,E_s,v_s,rho_s,R_s,h_s,d31,Vac_val);
C_ad = nd*C_ad;
M_ad = M_ad/nd;
da = nd*da;
R_ad = 2*zeta*sqrt(M_ad/C_ad);

freq = 10:10:2000;
results=zeros(length(freq),12);
phase_inc = 1;

for index_f = 1:length(freq)
    omega = 2*pi*freq(index_f); % Angular velocity [rad/s]
s = 1i*omega;
    Qd = omega*vol_change; % Max vol velocity displaced by diaphragm [m^3/s]

    phase = 0;
    u_real_max = 0;

    while phase < 360
        Vac = Vac_val*(cos(phase/180*pi)+1i*sin(phase/180*pi));
        voltage_angle = angle(Vac);
        Qo = Qd; % Initial guess of volume velocity through the orifice

        % C_ac - Acoustic compliance of cavity [m^4*s^2/kg]
        % M_al - Acoustic mass in the orifice [kg/m^4]
        % R_al - Acoustic resistance in the orifice [kg/(m^4*s)]
        % M_arad - Acoustic radiation mass [kg/m^4]
        % R_anl - Acoustic discharge resistance [kg/(m^4*s)]
        C_ac = Vc/(Rho*a_square);
        K = 1; % Dimensionless loss coefficient of orifice
        R_arad = 0; % Assumed negligible

        if setConfig == 1
            R_al = 128*Rho*nu*h/(pi*Do^4)/n;
            M_al = 16*Rho*h/(3*pi*Do^2)/n;
            M_arad = 16*Rho/(3*pi*Do^2)/n;
            R_anl = 8*K*Rho*Qo/(pi*2*Do^4)/n;
        elseif setConfig == 2
            R_al = 3*Rho*nu*h/(2*(Wo/2)^3*Bo)/n;
            M_al = 3*Rho*h/(5*Wo/2*Bo)/n;
            M_arad = Rho/(pi*Bo)*((Bo/2/Bo)+1/(2*(1-omega^2*Bo^2/a_square/6)))/n;
            R_anl = 0.5*K*Rho*Qo/(Wo^2*Bo^2)/n;
        elseif setConfig == 3
            R_aradc = 0;
            R_arads = 0;
            R_alc = 128*Rho*nu*h/(pi*Do^4);
            M_alc = 16*Rho*h/(3*pi*Do^2);
            M_aradc = 16*Rho/(3*pi*Do^2);
            R_als = 3*Rho*nu*h/(2*(Wo/2)^3*Bo);
            M_als = 3*Rho*h/(5*Wo/2*Bo);
            M_aradc = Rho/(pi*Bo)*((Bo/2/Bo)+1/(2*(1-omega^2*Bo^2/a_square/6)));
            R_anlc = 8*K*Rho*Qo/(pi*2*Do^4);
            R_anls = 0.5*K*Rho*Qo/(Wo^2*Bo^2);
            R_al = 1/(1/R_alc + 1/R_als);
            M_al = 1/(1/M_alc + 1/M_als);
            R_arad = 1/(1/R_aradc + 1/R_arads);
            M_arad = 1/(1/M_aradc + 1/M_arads);
            R_anl = 1/(1/R_anlc + 1/R_anls);
        else
            error('Error: Invalid setConfig');
        end

        a1 = C_ad*(R_ad + R_al+R_anl+R_arad) + C_ac*(R_al+R_anl+R_arad);
        a2 = C_ad*(M_ad + M_al+M_arad) + C_ac*(M_al+M_arad) +
             C_ac*C_ad*R_ad*(R_al+R_anl+R_arad);
        a3 = C_ac*C_ad*(M_ad*(R_al+R_anl+R_arad) + R_ad*(M_al+M_arad));
        a4 = C_ac*C_ad*M_ad*(M_al+M_arad);
        Qo_it = da*Vac*s/(a4*s^4+a3*s^3+a2*s^2+a1*s+1);

        while abs((Qo-Qo_it)/Qo) > 0.001
            Qo = Qo_it;
            if setConfig == 1
                R_anl = 8*K*Rho*Qo/(pi*2*Do^4)/n;
            elseif setConfig == 2
                R_anl = 0.5*K*Rho*Qo/(Do^2*Bo^2)/n;
            elseif setConfig == 3
                R_anl = 1/(1/R_anlc + 1/R_anls);
            end
        end
    end
end
else
    R_anlc = 8*K*Rho*Qo/(pi^2*Do^4);
    R_anls = 0.5*K*Rho*Qo/(Wo^2*Bo^2);
    R_anl = 1/(1/R_anlc + 1/R_anls);
end

a1 = C_ad*(R_ad + R_al+R_anl+R_arad) + C_ac*(R_al+R_anl+R_arad);
\n\na2 = C_ad*(M_ad + M_al+M_arad) + C_ac*(M_al+M_arad) +
    C_ac*C_ad*R_ad*(R_al+R_anl+R_arad);
\n\na3 = C_ac*C_ad*(M_ad*(R_al+R_anl+R_arad) + R_ad*(M_al+M_arad));
Qo_it = da*Vac*s/(a4*s^4+a3*s^3+a2*s^2+a1*s+1);
\nend
Qo = Qo_it;

if real(Qo) > u_real_max
    u_real_max = real(Qo);
end

phase = phase + phase_inc;

end

if setConfig == 1
    results(index_f,1) = u_real_max/(pi*Do^2)*4/n;
elseif setConfig == 2
    results(index_f,1) = u_real_max/(Bo*Wo)/n;
else
    results(index_f,1) = u_real_max/(Bo*Wo+(pi*Do^2)*4);
end

plot(freq,results(:,1));
xlabel('Frequency (Hz)');
ylabel('Velocity (m/s)');

\n\ndiaphragm.m function

function [da,dvol_change,C_ad,M_ad]=
PZT_diaphragm(E_p,v_p,rho_p,R_i,h_p,E_s,v_s,rho_s,R_o,h_s,d31,Vac)

z1_s = -h_s/2;
z2_s = z1_s + h_s;
z1_p = z2_s;
z2_p = z1_p + h_p;

Q_p = E_p/(1-v_p^2)*[1 v_p;v_p 1];
A_p = Q_p*(z2_p-z1_p);
B_p = Q_p*(z2_p^2-z1_p^2)/2;
D_p = Q_p*(z2_p^3-z1_p^3)/3;
Q_s = E_s/(1-v_s^2)*[1 v_s;v_s 1];
A_s = Q_s*(z2_s-z1_s);
B_s = Q_s*(z2_s^2-z1_s^2)/2;
D_s = Q_s*(z2_s^3-z1_s^3)/3;
\nA_i = A_p + A_s;
B_i = B_p + B_s;
D_i = D_p + D_s;
A_o = A_s;
B_o = B_s;
D_o = D_s;

alpha_i = B_i(1,1)/A_i(1,1);
Dstar_i = D_i(1,1)-B_i(1,1)^2/A_i(1,1);
alpha_o = B_o(1,1)/A_o(1,1);
Dstar_o = D_o(1,1)-B_o(1,1)^2/A_o(1,1);
\n\nsyms a1 i b1 i a1 o a2 o b1 o b2 o
for condition = 1:2
    Ef = Vac/h_p;
    if condition == 1
        P = 0;
    else
        Vac = 0;
        P = 1;
    end
    eq3 = a1_o*R_o + a2_o/R_o - alpha_o/Dstar_o*(P*R_o^3/16);
    eq4 = b1_o*R_o + b2_o/R_o - 1/Dstar_o*(P*R_o^3/16);
    eq5 = a1_i*R_i + alpha_i/Dstar_i*(P*R_i^3/16) - (a1_o*R_i + a2_o/R_i -
            alpha_o/Dstar_o*(P*R_i^3/16));
    eq6 = b1_i*R_i - 1/Dstar_i*(P*R_i^3/16) - (b1_o*R_i + b2_o/R_i -
            1/Dstar_o*(P*R_i^3/16));
    eq7 = A_i(1,1)*eps_rr_i + A_i(1,2)*eps_tt_i + B_i(1,1)*kap_rr_i +
          B_i(1,2)*kap_tt_i - Nr_p = (A_o(1,1)*eps_rr_o + A_o(1,2)*eps_tt_o + B_o(1,1)*kap_rr_o +
          B_o(1,2)*kap_tt_o);  
    eq8 = B_i(1,1)*eps_rr_i + B_i(1,2)*eps_tt_i + D_i(1,1)*kap_rr_i +
          D_i(1,2)*kap_tt_i - Mr_p = (B_o(1,1)*eps_rr_o + B_o(1,2)*eps_tt_o + D_o(1,1)*kap_rr_o +
          D_o(1,2)*kap_tt_o);
    S = solve(eq3,eq4,eq5,eq6,eq7,eq8,a1_i,b1_i,a1_o,a2_o,b1_o,b2_o);
    b1_ii = double(S.b1_i);
    b1_oo = double(S.b1_o);
    b2_oo = double(S.b2_o);
    syms r
    w_i = b1_i*(r^2-R_i^2)/2 + P*(r^4-R_i^4)/64/Dstar_i - (b1_oo*(r^2-R_i^2)/2 +
            b2_oo*log(R_o/R_i)) - P*(r^4-R_i^4)/64/Dstar_o;
    w_o = b1_oo*(r^2-R_o^2)/2 + b2_oo*log(r/R_o) - P*(r^4-R_o^4)/64/Dstar_o;
    vol_change = 2*pi*(int(-w_i*r,r,0,R_i) + int(-w_o*r,r,R_i,R_o));
    if (P == 0 && Vac == 0)
        C_ad = double(vol_change/P);
        M_ad = double(2*pi*(int((rho_p*h_p + rho_s*h_s)*w_i^2*r,r,0,R_i) +
            int(rho_s*h_s*w_o^2*r,r,R_i,R_o))/vol_change^2);
    elseif (P == 0 && Vac == 0)
        da = double(vol_change/Vac);
        dvol_change = double(vol_change);
    else
        error('Error: Invalid diaphragm parameters');
    end
end
APPENDIX B: CAD Drawings of Configurable Synthetic Jet Actuator

Diaphragm Clamp Plate (Outer)

Diaphragm Clamp Plate (Inner)

Figure B-1 CAD Drawing of Outer Diaphragm Clamp Plate

Figure B-2 CAD Drawing of Inner Diaphragm Clamp Plate
Cavity Plate (Single Circular Orifice)

Figure B-3 CAD Drawing of Singular Circular Orifice Cavity Plate

Cavity Plate (Double Circular Orifices)

Figure B-4 CAD Drawing of Double Circular Orifice Cavity Plate
Figure B-5 CAD Drawing of Mixed Orifice Cavity Plate

Figure B-6 CAD Drawing of Singular Slot Orifice Cavity Plate
Figure B-7 CAD Drawing of Double Slot Orifice Cavity Plate
APPENDIX C: Hot-Wire Anemometry

Temperature Sensitivity

In order to account for temperature variations throughout the experiment, the voltage value has to be corrected prior to linearization, by applying the ratio between calibration and measurement over-temperature,

\[ E_{corrected} = E \left( \frac{T_W - T_0}{T_W - T_{measured}} \right)^n \]  

where \( E_{corrected} \) is the corrected voltage value, \( E \) is the measured voltage, \( T_0 \) is the initial calibrated ambient temperature, and \( T_{measured} \) is the measured ambient temperature at the instance.

Velocity Calibration – Exponential Function

Based on King’s Law, a direct relationship can be prescribed between the voltage value and the flow velocity upon linearization. Figure E-1 shows the line fit adopted for velocity calibration.

![Figure C-1 Hot-Wire Calibration based on Exponential Function](image_url)
APPENDIX D: Damping Coefficient of Piezoelectric Diaphragm

To estimate the damping ratio of the piezoelectric diaphragm used in the SJA, a standard second order homogeneous linear ordinary differential equation (ODE) is given as

\[ m\ddot{x} + b\dot{x} + kx = 0 \]  (90)

under the assumption that mass \( m \) and spring constant \( k \) are both positive. Without a damping term, the ratio \( k/m \) is the square of the system’s natural angular frequency, and since damping ratio \( \zeta \) is the ratio of \( b/m \) to the critical damping constant \( 2\omega_n \), the ODE can be rewritten as

\[ \ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = 0 \]  (91)

The general solution is written as (87) and subsequently the damped natural frequency can be expressed as (88)

\[ x = e^{-\zeta \omega_n t} (A \cos \omega_d t - B \sin \omega_d t) \]  (92)

\[ \omega_d = \frac{2\pi}{t_2 - t_1} \]  (93)

By logarithmic decrement, the difference of natural logarithms of the two successive maxima of \( x \) is

\[ \Delta = \ln \frac{x_1}{x_2} \]  (94)

As values of \( \cos(\omega_d t - \phi) \) at two points of time differing by \( 2\pi/\omega_d \) are equal,

\[ \frac{x_1}{x_2} = e^{\zeta \omega_n (t_2 - t_1)} \]  (95)

\[ \Delta = \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}} \]  (96)
\[ \zeta = \frac{\left( \frac{\Delta}{2\pi} \right)}{\sqrt{1 + \left( \frac{\Delta}{2\pi} \right)^2}} \]  

(97)

In conducting the experiment to capture the damping response, the SJA was clamped and an accelerometer was attached to the piezoelectric disc. An impulsive force was applied directly onto the piezoelectric disc and the acceleration response was recorded.

For an underdamped case, where \( \zeta < 1 \),

\[
\ddot{x}(t) = (\zeta \omega_n)^2 \cdot e^{-\zeta \omega_n t} (A \cos \omega_d t + B \sin \omega_d t) - 2 \zeta \omega_n \omega_d \cdot e^{-\zeta \omega_n t} (-A \sin \omega_d t + B \cos \omega_d t) - \omega_d^2 \cdot e^{-\zeta \omega_n t} (A \cos \omega_d t + B \sin \omega_d t)
\]

(98)

Linearizing the above equation, smaller magnitude terms containing \( \zeta \) are neglected,

\[ x(t) \approx \frac{\ddot{x}(t)}{\omega_d^2} \]

(99)

The Table D-1 shows the measured accelerometer readings of first two successive peaks, as illustrated in Figure D-1.

![successive peaks](image)

**Figure D-1 Amplitude of Forced Response Acceleration versus Time**

<table>
<thead>
<tr>
<th>Peak</th>
<th>Accelerometer Reading</th>
<th>Approximate Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.64</td>
<td>(-0.64/\omega_d^2)</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.47</td>
<td>(-0.47/\omega_d^2)</td>
</tr>
</tbody>
</table>

Therefore applying (92), the damping coefficient of the system can be estimated as 0.05.
APPENDIX E: Experimental and LEM Results for Various Configurations

Figure E-1 Case 1 Experimental and LEM Results

Figure E-2 Case 2 Experimental and LEM Results

Figure E-3 Case 3 Experimental and LEM Results

Figure E-4 Case 4 Experimental and LEM Results
Figure E-5 Case 5 Experimental and LEM Results

Figure E-6 Case 6 Experimental and LEM Results

Figure E-7 Case 7 Experimental and LEM Results

Figure E-8 Case 8 Experimental and LEM Results
Figure E-9 Case 9 Experimental and LEM Results

Figure E-10 Case 10 Experimental and LEM Results

Figure E-11 Case 11 Experimental and LEM Results

Figure E-12 Case 12 Experimental and LEM Results
Figure E-13 Case 13 Experimental and LEM Results

Figure E-14 Case 14 Experimental and LEM Results

Figure E-15 Case 15 Experimental and LEM Results

Figure E-16 Case 16 Experimental and LEM Results